

One of the big difficulties students have in a problem that involves many objects is to successfully identify third law interaction pairs.

So today, I'd like to look at a problem which shows us how to think about that.

So what we're going to look at is third law interaction pairs.

Now, the problem I'd like to consider is the following.

Suppose we have a block one and another block two sitting on block one on a surface and our surface has friction.

And I'd like to push block one with a force F .

And now I'd like to use Newton's second law to determine what is the maximum force I can push that block two will not slip.

So I'm trying to find F_{\max} such that block two does not slip.

Now, how do we even begin to think about this?

Well, the first thing that we have to decide is when we apply Newton's second law, our first question will be, what is the system that we'll choose?

And in this problem, there are three separate ways to think about this.

Just to show you an example, so system a will be block one, system b will be block two, and system c will be block one and block two.

And given these different systems, I can address different questions and we'll see that as we develop this.

So let's start with block one and we'll start with breaking our problem down into block one and block two.

And let's draw the free-body diagrams on those blocks, trying to identify action-reaction pairs.

So let's begin with block one.

First off, we know there's gravitational force and the Earth is the other pair there, which we're not drawing.

There's friction between the surfaces.

So there's a friction between the ground and block one.

There is the normal force of the ground on block one.

Block two is sitting on block one so there's a normal force of block two on block one.

And finally, as you push block one, there's a friction force between block two and block one that's opposing the fact that block one is being pushed forward.

So we have a friction force here between blocks two and one.

Now, this friction down here is kinetic.

But if the blocks are moving together, this friction here is static.

So those are the free-body force diagrams on one.

I'll choose unit vectors in a moment.

Now, what about two?

So let's draw two.

Well, again, $m_2 g$ -- the Earth is the other element of the interaction pair.

And now, block one is pushing block two up.

So we have block one pushing block two up and notice our indices make it very easy to see that our first Newton's third law interaction pair is the normal force of contact between the two blocks.

Now, what else?

Here's the subtle thing is that this whole system will move to the right.

What's the force that's making block two move to the right?

Well, it's static friction.

So static friction from block one and block two-- this is the static friction-- is causing block two to move to the right.

And now we can see, again our third law interaction pair.

So in this problem, we have two third law pairs, this one and I'll connect the line there, and those are the third law pairs.

Now, we know by the third law that they're equal and opposite in magnitude.

We can identify f .

We can call this one N if we wanted just to save ourselves the problem of writing a lot of indices.

Once we've done that, we're now ready to apply Newton's second law.

We haven't yet figured out what the condition is that it will just slip.

We'll get to that.

But for the moment, we can now apply vector decomposition.

So we need to choose some unit vectors.

Because I'm pushing the system this way, it makes sense for me to choose my \hat{i} to the right.

So here, I'm going to choose \hat{i}_1 and \hat{j}_1 .

Now, over here, I could choose the same unit vectors, even though I'm thinking about this as a completely separate problem with its own coordinate system.

And I'll choose \hat{i}_2 and \hat{j}_2 .

But because these unit vectors are in the same direction, they're equal.

And they're both moving in the positive i directions and so I expect both a_1 and a_2 to be positive.

And now I on block one, I can write down $F_1 = m_1 a_1$.

And because we have two different directions, I'll separate out.

I like to call this my "scorecard." And now I look at the forces.

Oh, I missed the pushing force.

But that's an interesting exercise.

When I looked at this diagram, I saw I had two forces going this way.

I had no force acting that way.

I went back.

I checked my free-body diagram, and recognized that I forgot to put F in there-- always a good exercise to double-check your free-body diagrams before you apply Newton's laws.

So now in the x -direction, we have the pushing force minus the static friction minus-- we'll call this f_k -- minus the kinetic friction and that's equal to $m_1 a_1$.

Now in the vertical direction, we have the ground friction, $N_{\text{ground } 1}$, minus block two pushing down on block one minus the gravitational force.

And there is no acceleration in that direction.

And I double-check my free-body diagrams, I check my signs, and that looks right to me.

Now, for block two, I'll apply the same analysis.

F_2 equals $m_2 a_2$.

Separate out my two unit directions.

Notice even though these unit vectors are the same, I'm emphasizing that I'm talking about block two.

I could have chosen different coordinate systems if I wanted.

Now, I look at my free-body diagrams.

On block two.

I see that its static friction is the only one in the positive \hat{i} direction.

So I have F equals $m_2 a_2$.

And in the vertical directions, I have that my force between the blocks, the normal force between the blocks, minus gravity is 0.