

There are many physical problems in which a body undergoes motion about a central point.

And when that happens, there's a natural coordinate system to describe that motion, which in two dimensions is polar coordinates.

So let's consider the orbit of an object.

For instance, the best example is a circular orbit.

If we have a circular orbit of an object, there is a central point, which we'll call P.

Now given this type of motion with an object, it naturally makes sense to choose a coordinate system called polar coordinates.

The way that coordinate system works is as follows.

First off, we need to choose a reference angle.

And so we'll choose a horizontal line.

And we'll draw a ray.

And we'll show a direction of increasing reference angle θ .

In this example, θ will go from 0 to 2π .

Along with the reference angle, we have a distance from the central point.

And that distance we'll refer to as r .

So the coordinates of our point are r and θ .

Now the variable r is always greater than 0 and can go to infinity-- greater than 0.

So this is our polar coordinate system.

When you have a coordinate system, remember, at every point in space, there has to be unit vectors.

So at this point right here, how do we choose unit vectors for polar coordinates?

We always choose the unit vectors to point in the increasing direction of the coordinate.

Take the r -coordinate.

That increases radially outward.

So our unit vectors here will have a \hat{r} pointing radially outward.

What about the θ direction?

Tangential to the circle, in this particular case.

Because θ is increasing in this direction, we choose our tangential unit vector, which we're going to call $\hat{\theta}$, which is at right angles to \hat{r} , to point in the direction of increasing θ .

And so at this point, we now have a set of unit vectors.

Now one has to be very careful in polar coordinates for the following reason.

Suppose that you're at another point over here.

Now because we have two different points, let's just give some names to these points.

We'll call this s_1 and this point s_2 .

And the unit vector's over here where r_1 and θ_1 were.

When we're at the point s_2 , we have to choose unit vectors exactly the same way.

\hat{r}_2 points in the direction of increasing r , and $\hat{\theta}_2$ points in the direction of increasing θ .

So what we see in polar coordinates is that \hat{r}_1 is not equal to \hat{r}_2 .

Why are they not equal?

They both are unit vectors, so they both have the same magnitude, but they point in opposite directions, in the same way that $\hat{\theta}_1$ is not equal to $\hat{\theta}_2$.

So unlike Cartesian coordinates, in which at every single point had the same unit vectors, in polar coordinates, the unit vectors depend on where you are in space.

And that will make our analysis on polar coordinates a little bit more complicated.