

Let's look at a typical application of Newton's second law for a system of objects.

So what I want to consider is a system of pulleys and masses.

So I'll have a fixed surface here, a ceiling.

And from the ceiling, we'll hang a pulley, which I'm going to call pulley A. And this pulley will have a rope attached to it, wrapped around it.

And here we have object 1.

And the rope goes around the pulley.

And now it's going to go around another pulley B and fixed to the ceiling.

So that's fixed.

And hanging from pulley B is another mass 2.

And our goal in applying Newton's second law is to find the accelerations of objects 1 and 2.

Now, how do we approach this?

Well, the first thing we have to do is decide if we're going to apply Newton's second law, what is the system that we'll apply it to?

And there's many different ways to choose a system.

When we look at this problem, we'll have several different systems.

So let's consider the ones that we're going to look at.

And the first one is very simple.

It will be block 1.

And the second system that we look at, we'll call that AB is pulley A.

Now that brings us to an interesting question about pulley B and block 2, because we could separately look at pulley B, and we could separately consider block 2, or we consider them together.

And I want to first consider separately pulley B and block 2.

Now in some ways when you're looking at a compound system, and it has four objects, it makes sense to apply Newton's second law to each object separately.

But we'll pay careful attention to the fact that object 2 is connected to pulley B.

And eventually, we'll see that we can combine these two things.

So the next step is once we have identified our object is to draw a free body force diagrams for each of the objects.

So in order to do that, let's start with object 1.

And we want to consider the forces on object 1.

Now that brings us to our first issue about what types of assumptions we're making in our system.

For instance, we have a rope that's wrapped around this pulley.

And we have two pulleys that in principle could be rotating.

But what we'd like to do to simplify our analysis-- so let's keep track of some assumptions here.

Our first assumption will be that the mass, M_P , of pulley A and the mass of pulley B are approximately zero.

Now the reason for that is that we're not going to consider any of the fact that these objects have to be put into rotational motion.

Later on in the course, we'll see that this will give us a more complicated analysis.

We're also going to assume that our rope is not slipping.

So the rope is actually is just slipping on the pulleys.

So what that means is it's just the rope is sliding as the objects move.

Now again, what this is going to imply is that the tension in the rope-- this rope is also slipping.

And the rope is massless as well.

It's very light rope.

And all of these assumptions we've seen when we analyze ropes tell us that the tension T is uniform in the rope.

So that's our first assumption.

And we need to think about this before we even begin to think about the forces on the object.

And now we can draw our forces.

What do we have?

We have the gravitational force on object 1.

And now we can identify the tension pulling in the string, pulling object 1 up.

Now for every time we introduce a free body diagram, recall that we have to choose what we mean by positive directions.

And in this case, I'm going to pick a unit vector down, \hat{j}_1 down.

So that's my positive direction for force.

Now before I write down all of Newton's laws, I'll just write down our various force diagrams.

So for pulley A, I have two strings that are pulling it downwards.

So I have tension and tension.

And this string, I'm going to call that T_2 , is holding that pulley up.

So we have the force diagram.

Now I could write MAG , but we've assumed that the pulley is massless.

And again, I'll call \hat{j}_A down.

For object 2, let's do pulley B first.

Now what are the forces on pulley B?

I have strings on both sides, T . Pulley B is massless, so I'm not putting gravitational force.

And this string is pulling B downwards, so that's T_3 .

And again, we'll write \hat{j}_B downwards.

And finally, I have block 2.

So I'll draw that over here.

I'll write block 2.

In fact, let's say a little space here.

We'll have j hat B downwards.

Now block 2, what do we have there?

We have the string pulling up block 2, which we've identified as T3.

And we have the gravitational force on block 2 downward, $M_2 g$.

And there we have j hat 2.

So I've now drawn the free body diagram of the various objects.

And that enables me to apply Newton's second law for each of these objects.

So let's begin.

We'll start with object 1.

We have-- remember in all cases, we're going to apply F equals $m a$.

So for object 1, we have $m_1 g$ positive downward minus T is equal to $m_1 a_1$.

And that's our F equals $m a$ on object 1.

So sometimes we'll distinguish that the forces we're getting from our free body diagram.

And A is a mathematical description of the motion.

For block 2, we have $m_2 g$ minus T3 is equal to $m_2 a_2$.

And now for pulley A, we have $2T$ pointing downwards minus T2 going upwards.

And because pulley A is massless, this is zero even though pulley A may be-- it's actually fixed too.

So it's not even accelerating.

And what we see here is this equation-- I'm going to quickly note that it tells us that the string holding pulley 2 up, T_2 , is equal to $2T$.

So we can think of if, we want to know what T_2 is, we need to calculate T .

And finally we have B. And what is the forces on B?

We have T_3 minus $2T$.

And again pulley B is 0.

And so we see that T_3 is equal to $2T$.

Now if you think about what I said before about combining systems, if we combine pulley B in block 2, visually what we're doing is we're just adding these to free body diagram together.

When we have a system B and block 2.

Let's call this \hat{j} downwards.

And when we add these free body diagram together, you see that the T_3 is now internal force to the system.

It cancels in pair by Newton's second law.

And all we have is the two strings going up, so we have T and T . And we have the gravitational force downward.

And separately, when we saw that T_3 equals $2T$ and we apply it there, then if we consider a system B2, and look at our free body diagram, we have m_2g minus $2T$ -- and notice we have the same result their $2T$ equals $m_2 a_2$.

So in principle now-- and I'll outline our equations.

We have equation 1.

We have equation 2.

And in these two equations, we have three unknowns, T , A_1 , and A_2 , but only two equations.

And so you might think, what about this missing third equation here?

However, in this equation, we have a fourth unknown, T_3 .

And this equation is just relating to T and T_3 .

So in principle, we would have four unknowns and three equations.

Or if we restrict our attention to these two equations, we have three unknowns and two equations.

Are unknowns T , A_1 and A_2 .

These are our unknowns.

And now our next step is to try to figure out what is the missing condition that's relating the sum of these unknowns.

And that will be a constraint condition that we'll analyze next.