

The little prince is sitting on his little planet, and he's watching the planets go by.

And so suddenly, he's seeing three of them.

One, two, three.

And he wonders, hmm, what would the center of mass of these three planets be?

So let's calculate it.

We have one planet here that's three times the mass of this guy.

And then this one has half of m_1 .

And I've written this up there already.

And the center of mass, when we want to determine its coordinate-- it is a coordinate that depends on this coordinate system, and this origin here-- then we need to have the total mass of the system, because it's a mass-weighted coordinate.

And so the total mass is going to be $4.5 m_1$.

And if we want to calculate this position function of the center of mass, R_{cm} , that is the mass weight here.

And then we need the sum of all of our masses times our distances.

We're going to sum here over j from 1 to n .

And what that means is we need to now write out the sum for our three planets.

And we need to give this a radius here, so the r_1 would be going from here to here.

r_2 goes from here to there.

And r_3 from here to here.

You need to write them out in the \hat{i} direction and in the \hat{j} direction.

Let's add that here.

And then sum it all up and calculate our r .

So let's write this out.

We have m_1 , and we're going to have 35 and 10.

$35 \hat{i}$ plus $10 \hat{j}$ plus m_2 .

It's going to be 5 and 20.

5 and 20.

And then m_3 , we have 40 and 30.

And what we can do-- oh, and of course that has to be divided by my system mass.

And what we can do is we can concentrate first on the x component, and then on the y component.

Maybe we'll just continue here.

So we're going to have-- and we can plug it in all the m 's.

We'll do it for the x component first.

We're going to have m_1 , and then we have $35 \hat{i}$ plus here we're going to have $1/2 m_2$.

That is $3m_1 \hat{i}$ plus-- and for m_3 , we have $0.5 m_1 \hat{i}$.

that's in the \hat{i} direction.

And we'll have to divide that over our system mass.

And then we do the same for the y component.

So m_1 , 10 plus $3 m_1$, 20 plus $0.5 m_1 \hat{j}$.

And again, we have to divide this over our system mass.

So this boils down to-- hang on.

Let me redo this again.

Let me actually look at the answer first.

What do I have here?

70 and 85, OK.

So this boils to m_1 over the system mass.

And we have 70 in the \hat{i} plus 85 in the \hat{j} direction.

So the 70 comes from this term, the 80 comes from this.

And I put it back together, and now we need to plug in this one here.

And so we will get in the end of that R_{cm} equals m over 4.5 m .

So the m goes away, and we have a factor of $1/4.5$ here.

We'll divide this through, and we're going to have 15.5 in the \hat{i} direction, and 18.9 in the \hat{j} direction.

All right, so let's see where this fits on our graph here.

So 15 in the \hat{i} is somewhere here.

And 19 is almost 20, so it's going to be here, so about there.

So this is my R_{cm} , and this here is my center of mass of the system of these three little planets.

Of course, we used approximate math here for all the planets.

But if we look at the real numbers, imagine that this would be Earth, and it has one Earth mass.

And if this were Saturn, it would have something like 318 Earth masses.

And if this is Pluto, it would have 0.0025 Earth masses.

You will see that Saturn really holds all the weight.

And if we were to do this calculation with these numbers here, then our R_{cm} would-- and keeping this coordinate system in the arrangement of the planets, then it would go right into-- if here's the center, it would go right next to the center right over here, because Saturn just weighs so, so, so much more than Pluto and Earth together.