

Let's look at this rocket sled here.

It's gone on in the snow, but it wants to stop.

There are two little devices mounted on the sled, and they can eject gas.

And so the forward one is used to eject gas to make the sled stop.

We want to derive a relation for the differential between the speed of the sled and the differential of the mass of the rocket sled.

But before we do that with a rocket equation, we need to actually consider what else we know about this system.

Well, we know that the dry mass has a mass of m_0 .

The fuel mass is given also as m_0 .

And we know that, at time of t equals 0, the speed of the sled is v_0 .

We also know that, at a later time, t plus Δt , we have the sled here whose mass is now m of t plus Δt .

And we have this low mass parcel that has been ejected, so the gas.

And that has the mass of Δm_f , so the mass of the fuel.

We furthermore know that, relative to the sled, this little gas parcel is moving with a speed u .

The rocket equation says that my force-- my external force-- has two terms.

We have the mass of the rocket times the acceleration of the sled minus the differential here of the mass times the speed u .

And these are actually all vectors.

And so this describes this little gas parcel here, and this one describes rocket and we know that v equals \hat{v}_i .

\hat{v}_i is going in the right-- in the direction of motion, and u equals \hat{u}_i .

We also know that no external forces apply to the sled, so that's actually 0.

We can then turn this equation.

We can apply all of this.

We will get 0 equals $m r \frac{dv_r}{dt}$ minus $\frac{dm_r}{dt} u$.

We can bring this one on the other side, and we end up with the relationship that we wanted to obtain-- namely, an equation that contains the differentials of the speed of the rocket, that sled, and the differential of the mass.