

We just arrived this relation here-- the relation between the differential of the speed of the rocket sled and the differential of the mass of the rocket.

And we want to ultimately get the speed of the rocket.

So we have to apply a technique called separation of variables and then we want to integrate.

What we're going to do is, first we're going to divide-- multiply by dt so that falls away, and we are left with $m \frac{dv}{v} = -u \frac{dm}{m}$.

And we're going to shuffle the m onto the other side.

And we're left with $v \frac{dv}{v} = -u \frac{dm}{m}$ -- u is a constant-- so that goes up front, and then we have $v dv = -u \frac{dm}{m}$.

That's an equation that we can integrate now.

And we can do that.

And the tricky bit is that we need to take care of the integration limits here.

We have v and m , and actually these now are all primes.

So we have v going from v_0 .

That is our initial condition here $v(t=0) = v_0$.

And for the mass, we have m prime equals m_0 .

Actually, not quite m_0 , it's $2m_0$ because the initial mass of this is the dry mass and the fuel mass, so that's $2m_0$, so this is the initial mass.

And then we go to m prime equals m of v .

All right, so let's do that.

We're going to get $v - v_0 = -u \ln \frac{m}{2m_0}$, integrated gives us \ln , so \ln .

And then we can immediately do this here over $2m_0$.

And well, we ultimately want this, so this is the v of t , of course.

And then we got $v_0 + u \ln \frac{m}{2m_0}$ And that is our equation.

So what does this equation tell us?

M_r , the mass of the rocket is less later on than it was before, which means this term here is going to be less than 0, which means the velocity is our initial velocity minus something.

Which means we have a decrease in velocity, which means my sled will eventually come to a stop.