

Let's look at these two blocks here.

Block 1 has a mass of  $3m$  and Block 2 has a mass of  $1 m$ .

And they're colliding and sticking together.

And then they're going to move in this direction and eventually, slide onto this rough surface here.

What we want to figure out is, what was that initial velocity  $v_0$  with which the two blocks collided?

So up to this part here-- before this  $x$  equals  $0$ -- there's no external force acting on the system.

So conservation of momentum holds, which means that  $p_{\text{final}}$  equals  $p_{\text{initial}}$ .

So let's put this together.

We're going to have  $3m v_0$ .

That block goes in the positive  $\hat{i}$  direction.

Block 2 goes the opposite way, so we have  $m v_0$ .

And then at the end, when they're stuck together before here, we have to consider only this portion here that pertains to a frictionless surface.

We have  $4m v_{\text{final}}$ .

And so the  $m$ 's fall out.

This is  $2$ .

This is  $4$ .

So we'll see that the final velocity is  $v_0$  half.

Now, in order to get to this  $v_0$ , we also have to consider energy.

And in particular, we have to use the work energy principle here, because this joint block is going to slide onto this rough surface here.

So we can make use of the work energy principle, as we know what the initial conditions are at this point here, using the conservation of momentum from here in the first place.

So we want to use work equals delta K, the work energy principle.

Let's look at the work first.

Work is the integral of  $F \cdot dS$ .

And we're dealing with a one-dimensional problem here, so we can just write  $F_x dx$ .

And we want to have this run from 0 to  $d$ .

I should say here that this block eventually comes to rest at  $x$  equals  $d$ .

We're going to use that in a second.

So let's look at what this  $F$  is.

That's the frictional force here at work.

And actually, which direction does it go?

It opposes the motion  $f_k$ .

So we're going to get a minus sign here, integral  $f_k dx$ , again going from  $x$  prime equals 0 to  $x$  prime equals  $d$ .

Now what is this  $f_k$ ? We know that  $f_k$  equals  $\mu_k N$ .

And from doing a quick free body diagram, we'll see that we have  $4mg$  going down, and we have a normal force going up.

So we'll see that  $N$  equals  $4mg$ , which we can then plug in here.

And then that goes over there.

And the coefficient of kinetic friction is actually given to us.

So that nicely works out.

So we're going to have minus integral  $x$  prime 0,  $x$  prime equals  $d$ , the coefficient first,  $b x$  squared, and then  $N$   $4mg dx$ .

If we integrate that, we're going to get minus  $4bmg$ .

And we'll integrate this one, so that's  $x^3$  over 3.

And we're going to plug in this value here, so we're going to get a  $d$ , and that second term goes away.

So we are left with  $d^3$  here.

So this is our work.

And now we need to look at the change in kinetic energy between here and there.

We already know that this block is at rest at  $x$  equals  $d$ .

So if the velocity is 0, our kinetic energy will be 0, and so our  $K_{\text{final}}$  minus  $K_{\text{initial}}$  is going to be just minus initial.

And that's initial here, which is the final situation of our collision.

So we're going to look at the combined block.

So we'll have minus  $\frac{1}{2}$  and we have  $4mv_{\text{final}}^2$ .

We determined here already that the final velocity is the initial  $\frac{1}{2}$ , so have minus.

Here we're going to have  $2m$ , and now  $v_0^2$  over 4.

And that actually gives us minus  $\frac{1}{2} m v_0^2$ .

OK, so now we can put this all together.

We apply this work energy principle.

So we're going to get minus  $\frac{4}{3} bmg d^3$  equals minus  $\frac{1}{2} m v_0^2$ .

We'll see that this and this and this and this goes.

And so we're going to get  $v_0$  equals  $\frac{8}{3}$ , and then we have  $bgd^3$  and the square root of that.

So this is our initial velocity with which the two blocks collided initially, then eventually went onto this rough surface and came to rest here at  $x$  equals  $d$ .