

We're now going to introduce the concept of potential energy.

Let's begin by considering a system where a conservative force is acting.

So I'll consider a conservative force, which I'll call F_c .

So force set to force, the work integral is path independent.

So the work integral for this force is the integral of $F_c \cdot ds$, which is the path going from point A to point B.

And for a conservative force, this integral does not depend upon the path from A to B. It's independent of A and B.

So it depends only upon the endpoints.

So this is a path independent integral.

And since it depends only upon the endpoints, I can write it, since it's going to be an integral from point A to point B-- this integral must be equal to some function of the final point.

So some function of r_B minus some function of the initial point r_A . And to just get this integral from A to B, in our usual way of evaluating a definite integral, it's going to be equal to some function of r_B minus some function of r_A , since the integral depends only upon the endpoints.

Now, let's call this function-- I'm going to make a sort of funny choice here-- so let's call this function minus U as a function of position factor r .

And we'll see the reason for this funny choice of minus sign in just a moment.

So now with this definition, my work integral, which again, is the integral of $F_c \cdot ds$ from point A to point B is now minus U of r_B minus minus U of r_A . So in other words, that's minus U of r_B -- so minus minus gives me a plus U of r_A .

For shorthand, I can write that as minus U sub B plus U sub A.

And since we start out at point A and go to point B, notice that I can also write this as the negative of the change in U. Since the final value of U is U_B and the initial value is U_A , so this minus U_B plus U_A is equal to minus ΔU , the change in U as we go from the initial to the final position.

And note that in addition to that, given that this is the work integral, I can summarize that by writing that-- so I'll say, note that ΔU is equal to the negative of the work done going from point A to point B. Now, let's write the

work kinetic energy theorem using this newly introduced U function.

So the work kinetic energy theorem, which tells us that the work done, which we've seen is $-\Delta U$ is equal to the change in kinetic energy ΔK , which I can write as $K_B - K_A$. Or I could also write that as $\frac{1}{2} M V_B^2 - \frac{1}{2} M V_A^2$.

So this is just me stating that the work done on the system is equal to the change in kinetic energy.

And I can write the work in terms of my function U that I've introduced here to $U_B - U_A$.

So I'm going to rearrange this equation now-- basically the one involving U's and the one involving kinetic energies-- so that I have all the terms involving point A on one side and all the terms involving point B on the other side.

So rearranging, I get that at point A $\frac{1}{2} M V_A^2 + U_A$ is equal to at point B $\frac{1}{2} M V_B^2 + U_B$. Now, notice however, that there is nothing special about how I chose the points A and B. They're completely arbitrary.

So that means that this equation must be true for any points A and B.

And what that means is that each side must be equal to the same constant for any point in the system.

So in fact, we can write that $K + U$ for any point must be equal to some constant, which I'm going to call E_{mech} .

So K here is the kinetic energy.

U is my function that I introduced, and we're going to call it the potential energy.

And E_{mech} -- and remember, this E_{mech} here is a constant.

E_{mech} is something that we call the total mechanical energy.

Now, what we've done here is that we've shown that the total mechanical energy, which is the sum of the kinetic energy and the potential energy, is a constant under the action of a conservative force.

In other words, if we look at this equation and look at how it changes with time, the change in the kinetic energy, plus the change in the potential energy is equal to the change in the total mechanical energy.

And this is 0 for our conservative force.

So in other words, the change in kinetic energy is balanced by the change in potential energy, such that the sum is 0 when the force acting is conservative.

Now, we've now introduced the very important concept of the potential energy that is associated with the conservative force.

And we see that the change in the potential energy, the way we defined it, the change in potential energy is equal to the negative of the work integral for our conservative force going from point A to point B. Now in fact, it's actually only the change in the potential energy that has physical significance.

We'll be concerned with potential energy differences or changes.

The actual value of the potential energy itself doesn't matter.

We're free to choose any convenient reference point, or 0 point, for measuring the potential energy.

It's equivalent to choosing a coordinate origin when we're talking about positions.

Now, the potential energy change is related to the work done by conservative forces.

But we know that in general, work can also be done by non-conservative forces.

Although, that work by non-conservative forces will depend upon the path taken from point A to point B.

So in general, the total work is given by the sum of the conservative work-- the work done by conservative forces, which we can relate to a potential energy change, and the non-conservative work done.

And it's this total work that tells us what the change in the kinetic energy is.

Now we'll soon see that in the presence of non-conservative forces, the total mechanical energy, which is K plus U , is not a constant.