We'd like to examine the motion of two particles.

Here's particle 1.

And here's particle 2.

And each particle can have some motion.

In between them is the center of mass.

And what we'd like to do is figure out how to describe the motion of each of these particles with respect to the center of mass.

So let's choose some coordinate system.

Origin down here.

Here's our particle r1.

Here's our particle r2.

And here is our center of mass.

Now in the reference frame of the center of mass, we have position vector r1 prime.

And we have position vector r2 prime.

And what we'd like to do is find expressions for r2 prime and r2 prime in terms of the positions of r1 and r2.

Now we call that the position of the center of mass for this two body problem is given by m2 m1 r1 plus m2 r2 divided by the total mass.

And here we use the vector triangle that r1 prime equals r1 minus center of mass.

Now remember this vector is equal to this vector minus that vector.

Sometimes people like to say the vector r1 is equal to r plus r prime.

And that's how we get that relationship.

Now we can use our result here that r1 prime is r1 minus m1 r1 plus m2 r2 divided by m1 plus m2.

And when we combine terms-- let's just do this so you can see it-- r1 minus m1 r1 plus m2 r2 divided by the total mass.

We now have the m1 r1 terms cancel.

And we have a common m2 over m1 plus m2 times r1 minus r2.

Now r1 minus r2 is a vector that goes from-- here is r1.

Here's r2.

So the vector r1 minus r2 is the relative position of vector 1 with respect to 2.

And let's give that a special name.

We'll call that r1, 2, the relative position vector.

So we have m2 over m1 plus m2 r1, 2 is r1 prime.

Now you can easily see that if you interchange the indices 1 and 2, the only thing that changes here is a sign.

And if we interchange 1 and 2-- and this is an exercise that you can do-- then r2 prime is minus m1-- I'm interchanging the indices.

The minus sign came from the interchange of 1 and 2.

And so we get m1 over m1 plus m2 with the minus sign r1, 2.

Now what is the significance of this result?

If you know the position of r1 and r2, you know the relative velocity.

If you have information about this relative position, if you know the relative position vector, then you can separately get the locations of the two objects in the center of mass frame.

Now this quantity in here will appear often.

And I'd like to introduce a new quantity called the reduced mass.

And that reduced mass, mu, is the product of m1 m2 over m1 plus m2.

It's a simple exercise to see that 1 over mu is 1 over m1 plus 1 over m2.

And we'll encounter that a little bit later.

Then I can write both of these vectors-- and this is our conclusion-- that r1 prime is the reduced mass.

Notice we have an m2 here, so we have to divide by m1 times the vector r1, 2.

And r2 prime is minus the reduced mass.

Again, we now have to divide by m2.

And you see this nice symmetry of m1 and 1, 2 and 2, r1, 2.

And that's our key result.