Class 02: Outline

Answer questions

Hour 1:
  Review: Electric Fields
  Charge
  Dipoles

Hour 2:
  Continuous Charge Distributions
Last Time: Fields
Gravitational & Electric
Gravitational & Electric Fields

Mass $M$  Charge $q$ (±)

CREATE:

$$\vec{g} = -G \frac{M}{r^2} \hat{r}$$

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

FEEL:

$$\vec{F}_g = m \vec{g}$$

$$\vec{F}_E = q \vec{E}$$

This is easiest way to picture field
PRS Questions:
Electric Field
Electric Field Lines

1. Direction of field line at any point is tangent to field at that point
2. Field lines point away from positive charges and terminate on negative charges
3. Field lines never cross each other
Consider two point charges of equal magnitude but opposite signs, separated by a distance $d$. Point $P$ lies along the perpendicular bisector of the line joining the charges, a distance $s$ above that line. What is the E field at $P$?
Two PRS Questions:
E Field of Finite Number of Point Charges
Charging
How Do You Charge Objects?

- Friction
- Transfer (touching)
- Induction

+q

- - Neutral +
- +
- +
- +
Demonstrations: Instruments for Charging
Electric Dipoles

A Special Charge Distribution
Electric Dipole

Two equal but opposite charges $+q$ and $-q$, separated by a distance $2a$

\[ \vec{p} \equiv \text{charge} \times \text{displacement} \]
\[ = q \times 2a \hat{j} = 2qa \hat{j} \]

\[ \vec{p} \] points from negative to positive charge
Why Dipoles?

Nature Likes To Make Dipoles!

Dipoles make Fields
Electric Field Created by Dipole

Thou shalt use components!

\[ \frac{\vec{r}}{r^2} = \frac{\vec{r}}{r^3} = \frac{\Delta x \hat{i}}{r^3} + \frac{\Delta y \hat{j}}{r^3} \]

\[ E_x = k_e q \left( \frac{\Delta x}{r_+^3} - \frac{\Delta x}{r_-^3} \right) = k_e q \left[ \frac{x}{\left( x^2 + (y-a)^2 \right)^{3/2}} - \frac{x}{\left( x^2 + (y+a)^2 \right)^{3/2}} \right] \]

\[ E_y = k_e q \left( \frac{\Delta y_+}{r_+^3} - \frac{\Delta y_-}{r_-^3} \right) = k_e q \left[ \frac{y-a}{\left( x^2 + (y-a)^2 \right)^{3/2}} - \frac{y+a}{\left( x^2 + (y+a)^2 \right)^{3/2}} \right] \]
PRS Question:
Dipole Fall-Off
Point Dipole Approximation

Take the limit $r \gg a$

You can show...

\[
E_x \rightarrow \frac{3p}{4\pi\varepsilon_0 r^3} \sin \theta \cos \theta
\]

\[
E_y \rightarrow \frac{p}{4\pi\varepsilon_0 r^3} \left(3 \cos^2 \theta - 1\right)
\]
Shockwave for Dipole

Dipoles *feel* Fields
Demonstration: Dipole in Field
Dipole in Uniform Field

\[ \vec{E} = E \hat{i} \]
\[ \vec{p} = 2qa(\cos \theta \hat{i} + \sin \theta \hat{j}) \]

Total Net Force:  \[ \vec{F}_{\text{net}} = \vec{F}_+ + \vec{F}_- = q\vec{E} + (-q)\vec{E} = 0 \]

Torque on Dipole:  \[ \vec{\tau} = \vec{r} \times \vec{F} = \vec{p} \times \vec{E} \]
\[ \tau = rF_+ \sin(\theta) = (2a)(qE) \sin(\theta) = pE \sin(\theta) \]

\[ \vec{p} \] tends to align with the electric field
Torque on Dipole

Total Field (dipole + background) shows torque:


- Field lines transmit tension
- Connection between dipole field and constant field “pulls” dipole into alignment
PRS Question:
Dipole in Non-Uniform Field
Continuous Charge Distributions
Continuous Charge Distributions

Break distribution into parts:

\[ Q = \sum_{i} \Delta q_i \rightarrow \int_{V} dq \]

E field at \( P \) due to \( \Delta q \)

\[ \Delta \vec{E} = k_e \frac{\Delta q}{r^2} \hat{r} \rightarrow d\vec{E} = k_e \frac{dq}{r^2} \hat{r} \]

Superposition:

\[ \vec{E} = \sum \Delta \vec{E} \rightarrow \int d\vec{E} \]
Continuous Sources: Charge Density

Volume \( V = \pi R^2 L \)

\[ dQ = \rho \, dV \]
\[ \rho = \frac{Q}{V} \]

Area \( A = wL \)

\[ dQ = \sigma \, dA \]
\[ \sigma = \frac{Q}{A} \]

Length \( L \)

\[ dQ = \lambda \, dL \]
\[ \lambda = \frac{Q}{L} \]
Examples of Continuous Sources: Line of charge

Length = $L$

$dQ = \lambda \, dL$

$\lambda = \frac{Q}{L}$

Examples of Continuous Sources: Line of charge

Length = \( L \)

\[
dQ = \lambda \, dL
\]

\[
\lambda = \frac{Q}{L}
\]

Examples of Continuous Sources: Ring of Charge

\[ dQ = \lambda \, dL \]

\[ \lambda = \frac{Q}{2\pi R} \]

Examples of Continuous Sources: Ring of Charge

\[ dQ = \lambda \, dL \]

\[ \lambda = \frac{Q}{2\pi R} \]

Example: Ring of Charge

$P$ on axis of ring of charge, $x$ from center
Radius $a$, charge density $\lambda$.

Find $\mathbf{E}$ at $P$
1) Think about it

\[ E_\perp = 0 \quad \text{Symmetry!} \]


2) Define Variables

\[ dq = \lambda \, dl = \lambda (a \, d\varphi) \]

\[ r = \sqrt{a^2 + x^2} \]
Ring of Charge

3) Write Equation

\[ d\vec{E} = k_e dq \frac{\hat{r}}{r^2} = k_e dq \frac{\vec{r}}{r^3} \]

a) My way

\[ dE_x = k_e dq \frac{x}{r^3} \]

b) Another way

\[ dE_x = \left| d\vec{E} \right| \cos(\theta) = k_e dq \frac{1}{r^2} \cdot \frac{x}{r} = k_e dq \frac{x}{r^3} \]
4) Integrate

\[ E_x = \int dE_x = \int k_e \, dq \, \frac{x}{r^3} = k_e \frac{x}{r^3} \int dq \]

Very special case: everything except \( dq \) is constant

\[ \int dq = \int_0^{2\pi} \lambda a \, d\varphi = \lambda a \int_0^{2\pi} \, d\varphi = \lambda a 2\pi \]

\[ = Q \]
5) Clean Up

\[ E_x = k_e Q \frac{x}{r^3} \]

\[ E_x = k_e Q \frac{x}{(a^2 + x^2)^{3/2}} \]

\[ \vec{E} = k_e Q \frac{x}{(a^2 + x^2)^{3/2}} \hat{i} \]

6) Check Limit \( a \to 0 \)

\[ E_x \to k_e Q \frac{x}{(x^2)^{3/2}} = \frac{k_e Q}{x^2} \]
In-Class: Line of Charge

Point $P$ lies on perpendicular bisector of uniformly charged line of length $L$, a distance $s$ away. The charge on the line is $Q$. What is $E$ at $P$?
Typically give the integration variable ($x'$) a “primed” variable name.
E Field from Line of Charge

\[ \vec{E} = k_e \frac{Q}{s(s^2 + L^2 / 4)^{1/2}} \hat{j} \]

**Limits:**

\[ \lim_{s \gg L} \vec{E} \rightarrow k_e \frac{Q}{s^2} \hat{j} \]  
Point charge

\[ \lim_{s \ll L} \vec{E} \rightarrow 2k_e \frac{Q}{Ls} \hat{j} = 2k_e \frac{\lambda}{s} \hat{j} \]  
Infinite charged line
In-Class: Uniformly Charged Disk

$P$ on axis of disk of charge, $x$ from center
Radius $R$, charge density $\sigma$.

Find $\mathbf{E}$ at $P$
Disk: Two Important Limits

\[ \vec{E}_{\text{disk}} = \frac{\sigma}{2\varepsilon_0} \left[ 1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}} \right] \hat{i} \]

Limits:

\[ \lim_{x \gg R} \vec{E}_{\text{disk}} \rightarrow \frac{1}{4\pi\varepsilon_0} \frac{Q}{x^2} \hat{i} \quad \text{Point charge} \]

\[ \lim_{x \ll R} \vec{E}_{\text{disk}} \rightarrow \frac{\sigma}{2\varepsilon_0} \hat{i} \quad \text{Infinite charged plane} \]
E for Plane is Constant????

1) Dipole: E falls off like $1/r^3$
2) Point charge: E falls off like $1/r^2$
3) Line of charge: E falls off like $1/r$
4) Plane of charge: E constant