Class 17: Outline

Hour 1:
  Dipoles & Magnetic Fields

Hour 2:
  Expt. 7: Dipoles in B Fields
Last Time:
Biot-Savart
The Biot-Savart Law

Current element of length $ds$ carrying current $I$ produces a magnetic field:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{s} \times \hat{r}}{r^2}$$

Moving charges are currents too…

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \, \vec{v} \times \hat{r}}{r^2}$$

PRS Question:
Force between wires
Magnetic Dipoles: Torque & Force
First: Review From Friday
Rectangular Current Loop

Place rectangular current loop in uniform B field

\[ \vec{F}_1 = \vec{F}_3 = 0 \ (\vec{L} \parallel \vec{B}) \]

\[ \vec{F}_2 = I(-a\hat{j}) \times (B\hat{i}) = IaB\hat{k} \]

\[ \vec{F}_4 = I(a\hat{j}) \times (B\hat{i}) = -IaB\hat{k} \]

\[ \vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0 \]

No net force on the loop!!
Torque on Rectangular Loop

Recall: \[ \mathbf{\tau} = \mathbf{r} \times \mathbf{F} \]

\[ \mathbf{\tau} = \left( -\frac{b}{2} \hat{i} \right) \times \mathbf{F}_2 + \left( \frac{b}{2} \hat{i} \right) \times \mathbf{F}_4 \]

\[ = \left( -\frac{b}{2} \hat{i} \right) \times (IaB\hat{k}) + \left( \frac{b}{2} \hat{i} \right) \times (-IaB\hat{k}) \]

\[ = \frac{IabB}{2} \hat{j} + \frac{IabB}{2} \hat{j} = IabB\hat{j} \]

**Torque Direction:**
Thumb in torque direction, fingers rotate with object
Torque on Rectangular Loop

\[ \vec{\tau} = IAB\hat{j} \]

\[ \vec{A} = A\hat{n} = ab\hat{n}: \text{area vector} \]

\[ \hat{n} = +\hat{k}, \quad \vec{B} = B\hat{i} \]

\[ \vec{\tau} = I\vec{A} \times \vec{B} \]

Familiar? No net force but there is a torque
Magnetic Dipole Moment

Define Magnetic Dipole Moment:

\[ \vec{\mu} \equiv I\vec{A} \hat{n} \equiv I\vec{A} \]

Then:

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]

Analogous to \( \vec{\tau} = \vec{p} \times \vec{E} \)

\( \tau \) tends to align \( \mu \) with \( \vec{B} \)
Animation: Another Way To Look At Torque

External field connects to field of compass needle and “pulls” the dipole into alignment
Interactive Java Applet: Another Way To Look At Torque

Field of wire connects to field of compass needle and “pulls” the dipole into alignment

Demonstration: Galvanometer
Magnetic Dipole Moment

\[ \vec{\mu} \equiv I A \hat{n} \equiv I \vec{A} \]
PRS Question:
Torque on Dipole in Uniform Field
Dipoles don’t move???

This dipole rotates but doesn’t feel a net force.

But dipoles CAN feel force due to $\mathbf{B}$. What’s up?
PRS Question:
Force on Magnetic Dipole
Something New
Dipoles in Non-Uniform Fields: Force
Force on Magnetic Dipole

To determine force, we need to know energy:

\[ U_{Dipole} = -\vec{\mu} \cdot \vec{B} \]

Force tells how the energy changes with position:

\[ \vec{F}_{Dipole} = -\vec{\nabla} U_{Dipole} = \vec{\nabla} \left( \vec{\mu} \cdot \vec{B} \right) \]

(after math) \[ = \left( \vec{\mu} \cdot \vec{\nabla} \right) \vec{B} \]

Dipoles only feel force in non-uniform field
Force on Magnetic Dipole

\[ \vec{F}_{Dipole} = (\vec{\mu} \cdot \vec{\nabla}) \vec{B} = \mu \frac{\partial \vec{B}}{\partial z} \]

\( \mu = \text{NIA} \)

negative

Force down!
Force on Magnetic Dipole

Alternate Thought

What makes the field pictured?
Force on Magnetic Dipole

Bar magnet below dipole, with N pole on top
It is aligned with the dipole pictured, they attract!
Experiment 7: Magnetic Forces on Dipoles
Force on Dipole from Dipole: Anti-Parallel Alignment

Force on Dipole from Dipole: Parallel Alignment

PRS Questions:
Force on Magnetic Dipole