Assignment #12
Displacement Current, E/M Waves
Energy, Power, Momentum in E/M Waves
Transmission Lines

Reading Purcell: Chapter 9, Handouts on Electromagnetic (E/M) Waves, Polarization, Transmission Lines

Problem Set #12
Work on all problems. Not all problems receive equal points. Total points for this set is 100.


- (15 points) [2] Electromagnetic (E/M) plane waves.

For each of the following given $\vec{E}$ and $\vec{B}$ vectors (assuming they are describing an E/M plane wave) find (if exist) the accompanying $\vec{B}$ and $\vec{E}$ ones. Express your answer in terms of the given variables ($E_0$, $B_0$, $k$ and $\omega$ are positive definite constants). For each case also draw a plot showing a right handed Cartesian coordinate system with $x, y, z$ axes identified and with the vectors $\vec{E}$, $\vec{B}$ and $\vec{k}$ (propagation vector) shown on it. $\vec{E} = -E_0 \cos(kx + \omega t) \hat{x}$

$$\vec{B} = B_0 \cos(kz + \omega t) \hat{y}$$

$$\vec{E} = E_0 \sin(kx - \omega t) \hat{y}$$

$$\vec{B} = B_0 \sin(ky - \omega t) \hat{z}$$

- (20 points) [3] Coaxial cable and Poynting vector.

A coaxial cable "delivers" current $I = E/R$ from the emf $E$ to the resistor $R$ as shown in the figure. The coaxial cable is resistanceless and it is made up of an inner metallic conductor of radius $a$ and
an outer metallic conductor of radius $b$. Our goal is to extend the definition of the Poynting vector to static fields and show that its physical significance remains the same, i.e., a measure of power flow.

- Find the $\vec{E}$ and $\vec{B}$ fields in the space in between the two conductors of the coax cable and construct the Poynting vector $\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B}$. Where does $\vec{S}$ "poynt" to?

- Convince yourself this is the only region of space where $\vec{S}$ is non-zero.
- Integrate $\vec{S}$ over the cross section of the cable and show that the total power flowing through the cable is $E^2/R$. Is this what you expected?

- The leads of the battery are now reversed. Does the direction of $\vec{S}$ change? Is this what you expected?


  At the top of the atmosphere the average radiant flux from the Sun is $N = 1.35 \times 10^3 W/m^2$.

  Although this radiation consists of a spectrum of frequencies, many of the interesting properties do not depend on frequency and can therefore be calculated by using the methods described for monochromatic waves.

  - What is the average energy density in the solar radiation at the top of the atmosphere?
  - What is the average momentum density?
  - What average force would the radiation exert on a completely absorbing surface with an area of $1 m^2$ oriented perpendicular to the Earth-Sun line?
  - What is the average value of $E_0$ in the wave?

- (15 points) [5] $\vec{E}$ and $\vec{B}$ fields in a capacitor.

  We have worked in class on the $\vec{E}(\vec{r}, t)$ and $\vec{B}(\vec{r}, t)$ fields in a parallel circular plate capacitor (radius $R$, distance $l$) driven by an alternating current $I(t) = I_0 \cos(\omega t)$. In doing this we have ignored fringing effects, assumed $\vec{E}$ spatially uniform and also assumed $I$ being "slowly varying", i.e.,
\[ \omega R / 2\pi c \ll 1. \]

- Show that our assumption of uniform \( \vec{E} \) field is in violation of Faraday's law.
- Estimate the non-uniformity of \( \vec{E} \) by calculating the circulation of \( \vec{E} \) around the path shown in figure. Is \( \vec{E} \) going to be decreasing or increasing with increasing \( r \)?
- Find the relative error \( \Delta E / E \) and compare it with one of our assumptions (this saves us!).

- **(20 points) [6]** Wave Polarization.

An electromagnetic wave is the superposition of two linearly polarized wave along the \( \hat{y} \) and \( \hat{z} \) directions and is described by the following equation:

\[
\vec{E} = \hat{y} E_0 \sin(\omega t - \frac{\omega z}{c}) + \hat{z} E_0 \cos(\omega t - \frac{\omega z}{c})
\]

- What is the direction of propagation of the wave?
- What is the polarization status of this wave?
- Find the magnitude of the electric field at all points of space for all times.
- An observer stands at the origin of the coordinate system. Draw a diagram showing the vector \( \vec{E} \) at \( t = 0, t = \pi / 2\omega, t = \pi / \omega, t = 3\pi / 2\omega, t = 2\pi / \omega \).

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