Hints to Assignment #9 -- 8.022


We have worked on a similar problem in electrostatics by considering a hollow sphere. The name of the game here is superposition. In addition make sure you work in cylindrical coordinates and you do invoke the units vectors in this system.

- Assume that the current-free region results from the addition of a wire (radius a) carrying current \( J \) and another wire (of same radius a) carrying current \( -J \).
- Pick a random point \( P \) inside the hollow region and express the field there as the superposition of two wires, one of radius \( R \) carrying current \( J \) and another one of radius a carrying current \( -J \).
- There is one vector identity that is illuminating: \( \phi^\wedge = k^\wedge \times \rho^\wedge \) where these are the three unit vectors of the cylindrical system.


- Find the magnetic fields due to the two wires at any distance and invoke the superposition principle to find the total field across the loop.
- Perform a surface integral of \( B \) on the loop. Watch out, \( B \) changes with distance.
- \( \text{Emf} = -\frac{d\Phi}{dt} \) and all the \( d\Phi \) comes from the varying I's.
- Convince yourself that all currents induced are in agreement with Lenz's law!


Here are the steps that I see in this problem:

- Find (instantaneous) EMF from Faraday
- Find (instantaneous) current in circuit
- Find (instantaneous) magnetic force acting against the velocity
- Construct a differential eqn for the velocity using Newton's law- solve it with respect to \( u \) with the known initial conditions, \( u(t=0)=u_0 \), assume \( x=0 \) at \( t=0 \)
- If you know \( u(t) \) you can find \( x(t) \)
- From the instantaneous value of current \( I(t) \) you can find the power spent on the resistor and after integration the total energy spent on it. You should find \( \frac{m}{2} (u_0)^2 \) the initial kinetic energy of the rod!

You have all the basic ingredients to show this interesting feature of flux "deposition" on a loop:

- Use Faraday to find the EMF -- watch out the flux coupling is N times the flux of one loop.
- Find current in the loop \((I=\text{EMF}/R)\), insert it in Faraday
- Integrate both parts \(w/r/t\) time (from 0 when \(B_o\) to \(\tau\) when \(B=0\))


Very interesting problem as it shows that E/M field has angular momentum (and thus momentum) as well as how magnetic flux deposited on an atom may excite an electron in an atom (i.e., change its quantum numbers).

- Realize that azimuthal electric field sets up in order to make up for lost flux. \(\text{Int}(E\,dl)=-(1/c)\,d(\Phi)/dt\)
- E field exerts a force on q and thus a torque around loop's axis: \(\tau=Ra\,E\)
- Newton's law for rigid body motion: \(\tau=dL/dt\) where \(L\) is the angular momentum (work out directions of vectors prior to dealing with magnitudes) Integrate both sides \(w/r/t\) time to find out Delta \(L\).
- recall that \(L=I\,\omega\) where \(I\) is the moment of inertia \(\text{Int}(dm\,r^2)\) which for our charge \(q\) of mass \(m\) in circular motion will be \(mR^2\). Substitute in previous result to find \(\omega\).


The general strategy in finding mutual/self inductances is identifying the knows parts in the following two equations:

- Faraday: \(\text{EMF}=-\,(1/c)\,d(\Phi)/dt\) and \(\text{EMF}=-M\,dI/dt\) By equating the RHS you have an expression for \(M\). WATCH OUT in finding the flux linkage.
- Let's find the \(M_{12}\), i.e., the mutual inductance of the (inner) coil 1 due to the outer one (2). The equations from the above step read: \((1/c)\,d(\Phi_{12})/dt=M_{12}\,dI_2/dt\)
- Find \(\Phi_{12}\), i.e., the TOTAL flux on 1 due to field of 2. This will be \(\Phi_{12}=N_1*(4\pi/c)*(N_2/b_2)*I_2*(\pi*a_1^2)\). Pick it from here.
- It AIN'T OVER yet. Convince yourself by REPEATING the above steps that \(M_{21}=M_{12}\).
Coaxial conductors.

This is a very useful number for anyone wishing to improve his/her speaker's quality: self-inductance of the cable. You have two options:

- Find the flux coupling on the suggested rectangle and calculate how it varies as a function of time when current varies. Once you find $\Phi(t)$ you can find $\text{EMF} = -L \frac{di}{dt}$ and thus $L$. The integral for finding $\Phi$ is identical to problem [5]
- An alternative would be to find the energy of the magnetic field THROUGHOUT THE ENTIRE SPACE (there are three regions that this idealized coax creates) and then integrate to find the energy. Once you have the energy you can double it, divide by $I^2$ and thus find $L$. Is it the same as in the previous approach?

In approach #1 where we calculate the flux coupling, we arbitrarily select a rectangular loop hanging somewhere from the top of the wire and living in the space in between the conductor. But we could have hung INFINITE such loops all around the conductor's outer shell over a $2\pi$ geometry. Are we calculating the correct flux coupling? Do we have to imagine all such loops similar to the one suggested and add them up (i.e., by performing an integration over azimuth angle). Have we missed something?