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Solutions for Practice Quiz #6

The emf source $E$ is connected as shown in the figure below in a network that involves resistors $R_1$, $R_2$ and $R_L$.

Let us first introduce the arrows for $I_1, I_2, I_3, \text{emf}$ and the "positive" direction in summing potentials. Notice that with the exception of the emf, all other directions are completely arbitrary. Flipping the "positive" direction in summing potentials on a loop simply changes the sign of both hand sides of an equation. For what concerns $I_1, I_2$ and $I_3$ though, if we end up with negative values, that means that our initial guess of the direction was wrong and instead the opposite one is the correct.

There are 3 loops that are formed of which 2 equations are independent. There is also a junction equation (charge conservation). There are three unknowns ($I_1/I_2/I_3$) and we need three (independent) equations in order to determine them. Pick any 2 loop equations and the junction equation, follow Kirchhoff's laws and you are done.

$$E - I_1 R_1 - I_2 R_2 = 0$$
$$E - I_1 R_1 - I_3 R_1 - I_3 R_L = 0$$
$$I_1 = I_2 + I_3$$

Notice that there are 3 correct answers to this problem. Which are the other 2?
In calculating $R_{\text{eff}}$, proceed in steps identifying that you have the right most $R_1$ in series with $R_L$. Their sum is in parallel with $R_2$. This sum is in series with the left most $R_1$.

\[ R_{\text{eff}} = R_1 + \frac{(R_1+R_L)R_2}{R_1+R_L+R_2} \]

Equate the above $R_{\text{eff}}$ with $R_L$ to find out that:

\[ R_{\text{eff}} = R_L \Rightarrow R_1^2 + R_2R_1 + R_2^2 + R_4R_2 + R_LR_2 = R_4R_L + R_1^2 + R_2^2 \]
\[ \Rightarrow 2R_1R_2 = R_L - R_4^2 \Rightarrow R_2 = \frac{R_L - R_4^2}{2R_1} \]

In order to find out $V_{\text{CD}}$, you should identify that $I_1=\text{emf} / R_{\text{eff}}=\text{emf} / R_L$ (remember, $R_{\text{eff}}=R_L$) and $I_3=V_{\text{CD}} / R_L$. You then have:

\[ E - I_4R_4 - I_3(R_4+R_L) = 0 \Rightarrow E - \frac{E}{R_{\text{eff}}} R_4 - \frac{V_{\text{CD}}}{R_L} (R_4+R_L) = 0 \Rightarrow \]
\[ E - \frac{E}{R_L} R_4 - \frac{V_{\text{CD}}}{R_L} (R_4+R_L) = 0 \Rightarrow E (R_L - R_3) = V_{\text{CD}} (R_4 + R_L) \Rightarrow \]
\[ V_{\text{CD}} = \frac{E}{R_L} \frac{R_L - R_3}{R_L + R_3} \]