Problem 1

You come across a spherically symmetric electric field with the following form:

\[
\vec{E}(r) = \begin{cases} 
E_0 \left(\frac{r}{R}\right)^2 \hat{r} & 0 \leq r \leq R \\
E_0 \left(\frac{r}{R} - 2\right) \hat{r} & R < r \leq 2R \\
E_0 \left(\frac{3R}{r}\right)^2 \hat{r} & 2R < r \leq 3R \\
0 & r > 3R 
\end{cases}
\]

\(\hat{r}\) is the radial unit vector in spherical coordinates.

(a) For all \(r\), what is the charge \(Q(r)\) contained within a radius \(r\)?

(b) Calculate the charge density \(\rho(r)\) everywhere.

(c) Are there any surface charges in this charge distribution? If so, identify their location and give the magnitude of the surface charge density \(\sigma\) at each such location.

(d) The charge distribution is modified in some way. The new electric field is

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\end{cases}
\]

Compute the difference in energy between this and the old configuration, \(U_{\text{new}} - U_{\text{old}}\). Was work done on the system or did the system do work?
Problem 2

The circuit illustrated in the following figure is driven by an EMF $V = V_0 e^{i \omega t}$.

At time $t_1$, the switch is closed on A.

(a) Calculate the complex impedance $Z_{XY}$ of the circuit.

(b) What is the non-zero frequency $\omega_o$ that maximizes the average power dissipated in the resistor R?

3) For $\omega = \omega_o$, what is the average power dissipated in the resistor R?

4) Calculate $I_R$, the current flowing through the resistor R when $\omega = \omega_1 = 1/\sqrt{2CL}$. What is the phase between $I_R$ and V when $\omega = \omega_1 = 1/\sqrt{2CL}$? Is $I_R$ leading or lagging V?

At time $t_2$, the switch is closed on B.

5) What is the complex impedance $Z_{XY}$ of the circuit now?

6) How does $|V_R|$ depend on the frequency $\omega$? Draw a graph.

Problem 3

The electric field of a traveling electromagnetic wave in vacuum is given by $\vec{E} = 10(3\hat{x} + 4\hat{z}) \sin(ky + 2\pi 10^9t)$ statvolt/cm.

All given and requested numerical quantities are in CGS units.

(a) What is the direction of propagation of this wave?

(b) What is the frequency $f$ of this wave?

(c) What is the wavelength $\lambda$ of this wave?

(d) What is the amplitude of the electric field $E_o$?

(e) What is the amplitude of the corresponding magnetic field $B_o$?

(f) Find the corresponding magnetic field $\vec{B}$ to the given $\vec{E}$ above.

(g) What is the polarization of this wave?

(h) Find the Poynting vector $\vec{S}$ for this electromagnetic wave.

(i) A totally absorbing photodetector of flat surface A lies perpendicularly to the direction of propagation. What is the pressure the wave exerts as impinges on it? What is the intensity of the electromagnetic radiation collected by the photodetector?