Welcome to 8.022!

- 8.022: advanced electricity and magnetism for freshmen or electricity and magnetism for advanced freshmen?
- Advanced!
  - Both integral and differential formulation of E&M
  - Goal: look at Maxwell's equations

\[ \nabla \cdot \vec{E} = 4\pi \rho \quad \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \\
\nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \]

... and be able to tell what they really mean!
- Familiar with math and very interested in physics
- Fun class but pretty hard: 8.022 or 8.02T?
8.022 web page

Everything You Always Wanted to Know About 8.022 But Were Afraid to Ask...


Massachusetts Institute of Technology
Department of Physics

8.022 Electricity and Magnetism

Welcome to 8.022

General Course Information

News

Staff and Meetings

Lecturer: Prof. Gabriella Sciolia
Recitations: Prof. Erk Katavounidis

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<thead>
<tr>
<th>Lecture</th>
<th>Rec. Section #</th>
<th>Time</th>
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<tbody>
<tr>
<td>Prof. Sciolia</td>
<td>Tue &amp; Thu</td>
<td>9:30-11:00 AM</td>
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<tr>
<td>Prof. Katavounidis</td>
<td>Mon &amp; Wed</td>
<td>10-11 AM</td>
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<td>Prof. Katavounidis</td>
<td>Mon &amp; Wed</td>
<td>11-12 AM</td>
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<td>Prof. Katavounidis</td>
<td>Tue &amp; Thu</td>
<td>2-3 PM</td>
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<tr>
<td>Prof. Katavounidis</td>
<td>Tue &amp; Thu</td>
<td>3-4 PM</td>
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September 8, 2004  8.022 - Lecture 1
Textbook

E. M. Purcell
Electricity and Magnetism
Volume 2 - Second edition

- Advantages:
  - Bible for introductory E&M for generations of physicists

- Disadvantage:
  - cgs units!!!

Problem sets

- Posted on the 8.022 web page on Thu night and due on Thu at 4:30 PM of the following week
  - Leave them in the 8.022 lockbox at PEO

- Exceptions:
  - Pset 0 (Math assessment) due on Monday Sep. 13
  - Pset 1 (Electrostatics) due on Friday Sep. 17

- How to work on psets?
  - Try to solve them by yourself first
  - Discuss problems with friends and study group
  - Write your own solution
Grades

How do we grade 8.022?
- Homeworks and Recitations (25%)
- Two quizzes (20% each)
- Final (35%)
- Laboratory (2 out of 3 needed to pass)

NB: You may not pass the course without completing the laboratories!

More info on exams:
- Two in-class (26-100) quiz during normal class hours:
  - Tuesday October 5 (Quiz #1)
  - Tuesday November 9 (Quiz #2)
- Final exam
  - Tuesday, December 14 (9 AM - 12 Noon), location TBD

All grades are available online through the 8.022 web page.

September 8, 2004 8.022 - Lecture 1

...Last but not least...

Come and talk to us if you have problems or questions
- 8.022 course material
  - I attended class and sections and read the book but I still don’t understand concept xyz and I am stuck on the pset!
- Math
  - I can’t understand how Taylor expansions work or why I should care about them...
- Curriculum
  - is 8.022 right for me or should I switch to TEAL?
- Physics in general!
  - Questions about matter-antimatter asymmetry of the Universe, elementary constituents of matter (Sciolla) or gravitational waves (Kats) are welcome!
Your best friend in 8.022: math

- Math is an essential ingredient in 8.022
  - Basic knowledge of multivariable calculus is essential
  - You must be enrolled in 18.02 or 18.022 (or even more advanced)

- To be proficient in 8.022, you don’t need an A+ in 18.022
  - Basic concepts are used!

- Assumption: you are familiar with these concepts already but are a bit rusty...

Let’s review some basic concepts right now!

Derivative

- Given a function \( f(x) \), what is its derivative?
  \[
df = \frac{\partial f}{\partial x} \, dx
  \]
- The derivative \( \frac{\partial f}{\partial x} \) tells us how fast \( f \) varies when \( x \) varies.

\( \Rightarrow \) The derivative is the proportionality factor between a change in \( x \) and a change in \( f \).

- What if \( f = f(x,y,z) \)?
  \[
df = \frac{\partial f}{\partial x} \, dx + \frac{\partial f}{\partial y} \, dy + \frac{\partial f}{\partial z} \, dz
  \]

Gradient

Let's define the infinitesimal displacement \( d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z} \)

\[
df = \frac{\partial f}{\partial x} \, dx + \frac{\partial f}{\partial y} \, dy + \frac{\partial f}{\partial z} \, dz = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}) \cdot (dx, dy, dz) = \nabla f \cdot d\vec{l}
\]

Definition of Gradient:

\[
\text{grad } f = \nabla f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z} \equiv (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})
\]

Conclusions:
- \( \nabla f \) measures how fast \( f(x,y,z) \) varies when \( x, y \) and \( z \) vary
- Logical extension of the concept of derivative!
- \( f \) is a scalar function but \( \nabla f \) is a vector!
The “del” operator

Definition:
\[ \nabla \equiv \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = \left( \hat{x}, \hat{y}, \hat{z} \right) \]

Properties:
- It looks like a vector
- It works like a vector
- But it’s not a real vector because it’s meaningless by itself.
- It’s an operator.

How it works:
- It can act on both scalar and vector functions:
  - Acting on a scalar function: gradient \( \nabla f \) (vector)
  - Acting on a vector function with dot product: divergence \( \nabla \cdot \vec{f} \) (scalar)
  - Acting on a scalar function with cross product: curl \( \nabla \times \vec{f} \) (vector)

Divergence

Given a vector function \( \vec{v}(x, y, z) \)

\[ \vec{v}(x, y, z) \equiv v_x \hat{x} + v_y \hat{y} + v_z \hat{z} \equiv (v_x, v_y, v_z) \]

we define its divergence as:

\[ \text{div} \vec{v} \equiv \nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \]

Observations:
- The divergence is a scalar
- Geometrical interpretation: it measures how much the function \( \vec{v}(x, y, z) \) “spreads around a point”.

September 8, 2004  8.022 - Lecture 1  13
Divergence: interpretation

Calculate the divergence for the following functions:

\( \mathbf{v}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \)

\( \mathbf{v}(x, y, z) = z\mathbf{k} \)

\( \mathbf{v}(x, y, z) = -x\mathbf{i} - y\mathbf{j} - z\mathbf{k} \)

\[ \text{div } \mathbf{v} = 3 > 0 \text{ (faucet)} \]

\[ \text{div } \mathbf{v} = 0 \]

\[ \text{div } \mathbf{v} = -3 \text{ (sink)} \]

Does this remind you of anything?

Electric field around a charge has divergence ≠ 0!

\[ \text{div } \mathbf{E} > 0 \text{ for + charge: faucet} \]

\[ \text{div } \mathbf{E} < 0 \text{ for - charge: sink} \]
Given a vector function \( \vec{v}(x, y, z) \)

\[
\vec{v}(x, y, z) \equiv v_x \hat{x} + v_y \hat{y} + v_z \hat{z} \equiv (v_x, v_y, v_z)
\]

we define its curl as:

\[
\nabla \times \vec{v} \equiv \begin{vmatrix}
\hat{x} & \hat{y} & \hat{z} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
v_x & v_y & v_z
\end{vmatrix}
\]

Observations:

- The curl is a vector
- Geometrical interpretation: it measures how much the function \( \vec{v}(x, y, z) \) "curls around a point".

Calculate the curl for the following function:

\[
\vec{v}(x, y, z) = -y\hat{x} + x\hat{y}
\]

\[
\nabla \times \vec{v} = \begin{vmatrix}
\hat{x} & \hat{y} & \hat{z} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
-y & x & 0
\end{vmatrix} = 2\hat{k}
\]

This is a vortex: non zero curl!
Does this sound familiar?

Magnetic filed around a wire:

\[ \vec{\nabla} \times \vec{B} \neq 0 \]

An now, our feature presentation:
Electricity and Magnetism
The electromagnetic force:
Ancient history...

- 500 B.C. – Ancient Greece
  - Amber (ελεκτρόν = “electron”) attracts light objects
  - Iron rich rocks from μαγνεσία (Magnesia) attract iron
- 1730 - C. F. du Fay: Two flavors of charges
  - Positive and negative
- 1766-1786 – Priestley/Cavendish/Coulomb
  - EM interactions follow an inverse square law:
    \[ F_{em} \propto \frac{q_1 q_2}{r^2} \]
  - Actual precision better than 2/10^9!
- 1800 – Volta
  - Invention of the electric battery

N.B.: Till now Electricity and Magnetism are disconnected!

The electromagnetic force:
...History... (cont.)

- 1820 – Oersted and Ampere
  - Established first connection between electricity and magnetism
- 1831 – Faraday
  - Discovery of magnetic induction
- 1873 – Maxwell: Maxwell’s equations
  - The birth of modern Electro-Magnetism
- 1887 – Hertz
  - Established connection between EM and radiation
- 1905 – Einstein
  - Special relativity makes connection between Electricity and Magnetism as natural as it can be!
The electromagnetic force: Modern Physics!

- The Standard Model of Particle Physics
  - Elementary constituents: 6 quarks and 6 leptons

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Mediator</th>
<th>Relative Strength</th>
<th>Range (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>Gluon</td>
<td>$10^{37}$</td>
<td>$10^{-13}$</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>Photon</td>
<td>$10^{29}$</td>
<td>Infinite</td>
</tr>
<tr>
<td>Weak</td>
<td>$W^+/-, Z^0$</td>
<td>$10^{24}$</td>
<td>$10^{-15}$</td>
</tr>
<tr>
<td>Gravity</td>
<td>Graviton?</td>
<td>1</td>
<td>Infinite</td>
</tr>
</tbody>
</table>

The electric charge

- The EM force acts on charges
  - 2 flavors: positive and negative
    - Positive: obtained rubbing glass with silk
    - Negative: obtained rubbing resin with fur
- Electric charge is quantized (Millikan)
  - Multiples of the $e = \text{elementary charge}$
    - $e = 1.602 \times 10^{-19}$ C (SI), $4.803 \times 10^{-10}$ esu (cgs)
    - $Q_{\text{electron}} = -e$, $Q_{\text{proton}} = +e$
- Electric charge is conserved
  - In any isolated system, the total charge cannot change
    - If the total charge of a system changes, then it means the system is not isolated and charges came in or escaped.
Coulomb’s law

\[ \vec{F}_2 = k \frac{q_1 q_2}{| \vec{r}_{21} |^2} \hat{r}_{21} \]

- Where:
  - \( \vec{F}_2 \) is the force that the charge \( q_2 \) feels due to \( q_1 \)
  - \( \hat{r}_{21} \) is the unit vector going from \( q_1 \) to \( q_2 \)

- Consequences:
  - Newton’s third law: \( \vec{F}_2 = -\vec{F}_1 \)
  - Like signs repel, opposite signs attract

Units: cgs vs SI

- Units in cgs and SI (Sisteme Internationale)

<table>
<thead>
<tr>
<th></th>
<th>cgs</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>cm</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>g</td>
<td>Kg</td>
</tr>
<tr>
<td>Time</td>
<td>s</td>
<td>s</td>
</tr>
<tr>
<td>Charge</td>
<td>electrostatic units (e.s.u.)</td>
<td>Coulomb (C)</td>
</tr>
<tr>
<td>Current</td>
<td>e.s.u./s</td>
<td>Ampere (A)</td>
</tr>
</tbody>
</table>

- In cgs the esu is defined so that \( k=1 \) in Coulomb’s law \( \Rightarrow \)
  
  \[ 1 \text{ dyne} = \frac{(1\text{ esu})^2}{(1\text{ cm})^2} \Rightarrow 1 \text{ esu} = cm \sqrt{\text{dyne}} \]

- In SI, the Ampere is a fundamental constant
  - \( k=1/(4\pi\varepsilon_0)=8.99 \times 10^9 \text{ N C}^{-2} \text{ m}^2 \)
  - \( \varepsilon_0=8.85\times10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^2 \) is the permittivity of free space
Practical info: cgs - SI conversion table

<table>
<thead>
<tr>
<th>SI Units</th>
<th>CGS units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>1 Joule</td>
</tr>
<tr>
<td>Force</td>
<td>1 Newton</td>
</tr>
<tr>
<td>Charge</td>
<td>1 Coulomb</td>
</tr>
<tr>
<td>Current</td>
<td>1 Ampere</td>
</tr>
<tr>
<td>Potential</td>
<td>$3 \times 10^9$ Volts</td>
</tr>
<tr>
<td>Electric field</td>
<td>$3 \times 10^4$ Volts/m</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>1 Tesla</td>
</tr>
<tr>
<td>Capacitance</td>
<td>1 Farad</td>
</tr>
<tr>
<td>Resistance</td>
<td>$9 \times 10^14$ Ohm</td>
</tr>
<tr>
<td>Inductance</td>
<td>$9 \times 10^14$ Henry</td>
</tr>
</tbody>
</table>

FAQ: why do we use cgs?
- Honest answer: because Purcell does...

The superposition principle: discrete charges

The force on the charge $Q$ due to all the other charges is equal to the vector sum of the forces created by the individual charges:

$$\mathbf{F}_Q = \frac{q_1 Q}{|\mathbf{r}_1|^2} \mathbf{r}_1 + \frac{q_2 Q}{|\mathbf{r}_2|^2} \mathbf{r}_2 + \ldots + \frac{q_N Q}{|\mathbf{r}_N|^2} \mathbf{r}_N = \sum_{i=1}^{N} \frac{q_i Q}{|\mathbf{r}_i|^2} \mathbf{r}_i$$
The superposition principle: continuous distribution of charges

What happens when the distribution of charges is continuous? Take the limit for \( q_i \to dq \) and \( \Sigma \to \text{integral}:

\[
\vec{F}_Q = \sum_{i=1}^{N} \frac{q_i Q}{|r_i|^2} \hat{r}_i \to \int_{V} dq \frac{Q}{|r|^2} \hat{r} = \int_{V} \rho \frac{dV}{|r|^2} \hat{r}
\]

where \( \rho = \text{charge per unit volume: "volume charge density"} \)

The superposition principle: continuous distribution of charges (cont.)

- Charges are distributed inside a volume \( V \):
  \[
  \vec{F}_Q = \int_{V} \frac{\rho \ dV \ Q}{|r|^2} \hat{r}
  \]

- Charges are distributed on a surface \( A \):
  \[
  \vec{F}_Q = \int_{A} \frac{\sigma \ da \ Q}{|r|^2} \hat{r}
  \]

- Charges are distributed on a line \( L \):
  \[
  \vec{F}_Q = \int_{L} \frac{\lambda \ dl \ Q}{|r|^2} \hat{r}
  \]

Where:
- \( \rho = \text{charge per unit volume: "volume charge density"} \)
- \( \sigma = \text{charge per unit area: "surface charge density"} \)
- \( \lambda = \text{charge per unit length: "line charge density"} \)
**Application: charged rod**

P: A rod of length $L$ has a charge $Q$ uniformly spread over it. A test charge $q$ is positioned at a distance $a$ from the rod’s midpoint.

Q: What is the force $F$ that the rod exerts on the charge $q$?

![Diagram of a charged rod with a test charge positioned at a distance $a$ from the midpoint.]

**Answer:**

$$F = \frac{Qq}{a\sqrt{a^2 + \left(\frac{L}{2}\right)^2}}\hat{y}$$

**Solution: charged rod**

- Look at the symmetry of the problem and choose appropriate coordinate system: rod on $x$ axis, symmetric wrt $x=0$; $a$ on $y$ axis:

  ![Diagram showing symmetry and coordinate system for charged rod problem.]

  - Symmetry of the problem: $F \parallel y$ axis; define $\lambda = Q/L$ linear charge density
  - Trigonometric relations: $x/a = \tan \theta$; $a = r \cos \theta \Rightarrow dx = dr/a\cos\theta$; $r = a/\cos\theta$
  - Consider the infinitesimal charge $dF$, produced by the element $dx$:
    $$dF = dF \cos \theta = \frac{\lambda dx}{r} = \frac{Q}{a \cos \theta} $$
    $$= \frac{ad\theta}{a}$$
    $$= \frac{dq}{a}$$
  - Now integrate between $-L/2$ and $L/2$:
    $$F = \hat{y} \int_{-L/2}^{L/2} \frac{Qq}{a\sqrt{a^2 + \left(\frac{L}{2}\right)^2}} d\theta$$

  $$= \hat{y} \frac{Qq}{a\sqrt{a^2 + \left(\frac{L}{2}\right)^2}}$$
### Infinite rod? Taylor expansion!

Q: What if the rod length is infinite?

P: What does "infinite" mean? For all practical purposes, infinite means >> than the other distances in the problem: $L >> a$.

Let’s look at the solution:

$$F = \frac{Qq}{a \sqrt{a^2 + \left( \frac{L}{2} \right)^2}}$$

Taylor expand using $(2a/L)^2$ as expansion coefficient remembering that

$$(1 \pm x)^n = 1 \pm \frac{ax}{n!} + \frac{n(n-1)x^2}{2!} \pm ... \text{ for } x < 1$$

and

$$(1 \pm x)^n = 1 \pm \frac{ax}{n!} - \frac{n(n+1)x^2}{2!} \pm ... \text{ for } x > 1$$

$$\Rightarrow \quad F = \frac{\lambda L q}{a \left( 1 + \frac{2a}{L} \right)^2} = \frac{\lambda q}{2a} \left( 1 + \left( \frac{2a}{L} \right)^2 \right)^{-\frac{1}{2}} \approx \frac{\lambda q}{2a}$$

Rusty about Taylor expansions?

Here are some useful reminders...

**Exponential functions and natural logarithms:**

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{for all } x$$

$$\ln(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} \quad \text{for } |x| < 1$$

**Geometric series:**

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x| < 1$$

**Trigonometric functions:**

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \quad \text{for all } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \quad \text{for all } x$$