Complex impedance instead of diff eq!
Use fact that everything in RLC circuit has same frequency as driving frequency(?).

\[ V(t) = \hat{V} e^{i\omega t} \quad I(t) = \hat{I} e^{i\omega t} \]

**Inductor**

\[
\begin{align*}
V &= L \frac{dI}{dt} \\
\frac{dI}{dt} &= i\omega I \\
\frac{V}{I} &= i\omega L = \chi_L \quad \text{(complex impedance of inductor)}
\end{align*}
\]

**Capacitor**

\[
\frac{dV}{dt} = \frac{dQ}{dt} = \frac{I}{C} \\
\frac{dV}{dt} = i\omega V = \frac{I}{C}
\]

Complex impedance of capacitor

\[
\frac{V}{I} = \frac{1}{i\omega C} = \chi_C
\]

**Resistor**
\[ \frac{V}{I} = R \]

\[ V = I \cdot z \]

\[ \chi_L = i\omega L \]
\[ \chi_C = \frac{1}{i\omega C} \]
\[ \chi_R = R \]

**RLC Circuit**

No derivatives any more! Can sum just like resistors in series.

\[ \chi_{\text{total}} = \chi_R + \chi_C + \chi_L = R + i\omega L + \frac{1}{i\omega C} \]

\[ I = \frac{V}{\chi_{\text{total}}} = \frac{V}{R + i(\omega L - \frac{1}{i\omega C})} \cdot \frac{R - i(\omega L - \frac{1}{i\omega C})}{R - i(\omega L - \frac{1}{i\omega C})} = \frac{V(R - i[\omega L - \frac{1}{i\omega C}])}{R^2 + (\omega L - \frac{1}{i\omega C})^2} \]
\[
\hat{I} = \frac{V}{\left[R^2 + (\omega L - \frac{1}{\omega C})^2\right]^{1/2}} \\
\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R}
\]

Parallel RLC Circuit

Let \( Y = \frac{1}{\chi} \), \( I = V \cdot Y \)

admittance current

\[
Y_L = \frac{1}{i\omega L} \\
Y_C = i\omega C \\
Y_R = \frac{1}{R}
\]

\[
I = V \left( \frac{1}{R} + i(\omega C - \frac{1}{\omega L}) \right)
\]

\[
\hat{I} = V \left( \frac{1}{R^2} + (\omega C + \frac{1}{\omega L})^2 \right)^{1/2} \\
\tan \phi = R\omega C - \frac{R}{\omega L}
\]

Large \( \omega \): \( \frac{1}{L} \), \( V \omega C \) is important.
Small \( \omega \): no \( C \), \( \frac{V}{\omega L} \) important.

Can we do equivalent of Thevenin’s?
\[ \frac{V}{I} = \chi_{\text{effective}} \]

\[ z_{\text{eff}} = R_{\text{eff}} + i\chi_{\text{eff}} \]

First term decays, second term oscillates.

**Power Dissipation**

R does this! (LC circuit just oscillates, even w/o driver no loss of power).

\[ \frac{dV}{dt} = RI^2 \quad (= VI) \]
\[ z = R = i\chi \]

\[ z = i\chi \]
\[ V = i\chi I \]
\[ \hat{V}e^{i\omega t} = \chi\hat{I}e^{i\omega t + \frac{\pi}{2}} \]

\[
< P >_{\text{avg}} = \frac{1}{T} \int_{0}^{T} V \cdot I dt \\
= \int_{0}^{T} \hat{I}^2 R \cos^2(\omega t) dt - \frac{1}{T} \int_{0}^{T} \chi\hat{I}^2 \cdot \cos \omega t \sin \omega t dt \\
= \frac{\hat{I}^2 R}{2}
\]

**Ladder Impedence**

\[ z = z_1 + \frac{z_2 z}{z_2 + z} \]

Solve:

\[ z = \frac{z_1}{\hat{z}} + \sqrt{\frac{z_1^2}{4} + z_1 z_2} \]
Let:

\[ z_1 = i\omega L \]
\[ z_2 = \frac{1}{i\omega C} \]

\[ z = \frac{i\omega L}{2} + \sqrt{\frac{-\omega^2 L^2}{4} + \frac{L}{C}} \]

\[ v < 0 \quad \text{for} \quad \frac{\omega^2 L^2}{4} > \frac{L}{C} \]
\[ \omega^2 > \frac{4}{LC} \]

- for \( \omega^2 < \frac{4}{LC} \), there’s a real part = resistance! But from only \( L = C \)? It’s because its infinite! Energy keeps traveling out for certain \( \omega \)!

Critical Frequency - if you are under, energy will just keep going out. Otherwise, will go out and come back.