Schrodinger Equation
- matter waves
- probability wave

\[
i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi(x, t)\]

\[
P(x, t) = |\psi(x, t)|^2 = \psi^*(x, t) \psi(x, t)\]

\[
\frac{\partial}{\partial t} P(x, t) = \frac{\partial \psi^*}{\partial t} \psi + \psi^* \frac{\partial \psi}{\partial t} = -\frac{\partial}{\partial x} \left( \frac{\hbar}{2im} (\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \psi) \right)\]

Let \( j(x, t) = \frac{\hbar}{2im} (\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \psi) \),

\[
\frac{\partial}{\partial t} P + \frac{\partial}{\partial x} j = 0 \quad \text{Conservation law for probability}\]

\[
\frac{d}{dt} \int_a^b P(x, t) dx = j(b, t) - j(a, t)\]

Can write as:

\[
i\hbar \frac{\partial}{\partial t} \psi = \hat{H} \psi , \; \hat{H} = \text{Hamiltonian Operator}\]

Hamiltonian operator is an energy operator
\[ H = \frac{p^2}{2m} + V(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \]

First term is energy from momentum, second term is energy is potential energy

On operators:

\[ <\psi|\psi> = \int_{-\infty}^{\infty} dx \psi^* \psi \quad \hat{p} = \frac{\hbar}{i} \frac{\partial}{\partial x} \]
\[ <\psi|x|\psi> = \int_{-\infty}^{\infty} dx \psi^* x \psi \quad \hat{x} = x \]

If \( H \) is constant, then let’s call it \( E \) for certain \( t \)

\[ -\frac{\hbar}{2m} \cdot (\frac{\partial}{\partial x})^2 \psi(x) + V(x) \psi(x) = E \cdot \psi(x) \]

Take \( V(x) \) in the infinite square well case:

\[ E > 0 \text{ in general, } E < 0 \text{ unlikely} \]

\[ \frac{\partial^2}{\partial x^2} \psi + k^2 \psi = 0 \]
\[ k^2 = \frac{2mE}{\hbar^2} \]

\[ \psi = A \sin(kx) \]

\[ ka = n\pi \]

\[ E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \]

\[ \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \]