Problem Solving 3: Calculating the Electric Field of Highly Symmetric Distributions of Charge Using Gauss’s Law

REFERENCE: Section 4.2, 8.02 Course Notes.

Introduction

When approaching Gauss’s Law problems, we described a problem solving strategy summarized below (see also, Section 4.7, 8.02 Course Notes):

Summary: Methodology for Applying Gauss’s Law

Step 1: Identify the ‘symmetry’ properties of the charge distribution.

Step 2: Determine the direction of the electric field, and a surface on which the magnitude of the electric field is constant.

Step 3: Decide how many different regions of space the charge distribution determines

Step 4: For each region of space, choose the Gaussian surface such that the flux integral is either case a) or case b) above.

Step 5: Calculate the flux through the Gaussian surface.

Step 6: For each region of space, calculate the charge enclosed in the choice of the Gaussian surface for that region.

Step 7: For each region of space, equate the two sides of Gauss’s Law in order to find an expression for the magnitude of the electric field in that region of space.

Step 8: Graph the magnitude of the electric field as a function of the parameter specifying the Gaussian surface for all regions of space.

You should now apply this strategy to the following problem.
Question: Concentric Cylinders

A long very thin non-conducting cylindrical shell of radius $b$ and length $L$ surrounds a long solid non-conducting cylinder of radius $a$ and length $L$ with $b > a$. The inner cylinder has a uniform distribution of charge $\rho = \frac{Q}{L\pi a^2}$ distributed throughout its volume, where $+Q$ is the total charge on the inner cylinder. On the outer cylinder we place an equal and opposite to charge, $-Q$. The region $a < r < b$ is empty.

**Step 1 Question:** *(Answer on the tear-sheet at the end!)* What is the ‘symmetry’ property of the charge distribution here?

**Step 2 Question:** *(Answer on the tear-sheet at the end!)* What is the direction of the electric field, and what is a surface on which the magnitude of the electric field is constant?

**Step 3 Question:** *(Put your answer on the tear-sheet at the end!)* How many different regions of space does the charge distribution determine?

**Step 4 Question:** *(Put your answer on the tear-sheet at the end!)* For each region of space, describe your choice of the Gaussian surface. What variable did you choose to parameterize your Gaussian surface? What is the range of that variable?
Step 5 Question: *(Put your answer on the tear-sheet at the end!)* For the region for $r < a$, calculate the flux through your choice of the Gaussian surface. Your expression should include the unknown electric field for that region.

Step 6 Question: *(Put your answer on the tear-sheet at the end!)* For the region for $r < a$, calculate the charge enclosed in your choice of the Gaussian surface.

Step 7 Question 1: *(Put your answer on the tear-sheet at the end!)* For the region for $r < a$, equate the two sides of Gauss’s Law that you calculated in steps 5 and 6, in order to find an expression for the magnitude of the electric field.

Step 7 Question 2: *(Put your answer on the tear-sheet at the end!)* Repeat the same procedure in order to calculate the electric field as a function of $r$, the distance from the axis of the cylinders for the regions $a < r < b$. 
Step 8 Question: *(Put your answer on the tear-sheet at the end!)* Make a graph in the space below of the magnitude of the electric field as a function of the parameter specifying the Gaussian surface for all regions of space.

Penultimate Question *(Put your answer on the tear-sheet at the end!)* What is the potential difference between $r = a$ and $r = 0$? That is, what is $\Delta V = V(a) - V(0)$?

Final Question: *(Put your answer on the tear-sheet at the end!)* What is the potential difference between $r = b$ and $r = a$? That is, what is $\Delta V = V(b) - V(a)$?
Problem Solving 3: Using Gauss’s Law

Step 1 Question: What is the ‘symmetry’ property of the charge distribution here?

Step 2 Question: What is the direction of the electric field, and what is a surface on which the magnitude of the electric field is constant?

Step 3 Question: How many different regions of space does the charge distribution determine?

Step 4 Question: For each region of space, describe your choice of the Gaussian surface. What variable did you choose to parameterize your Gaussian surface? What is the range of that variable?
**Step 5 Question:** For the region for $r < a$, calculate the flux through your choice of the Gaussian surface. Your expression should include the unknown electric field for that region.

**Step 6 Question:** For the region for $r < a$, calculate the charge enclosed in your choice of the Gaussian.

**Step 7 Question 1:** For the region for $r < a$, equate the two sides of Gauss’s Law that you calculated in steps 5 and 6, in order to find an expression for the magnitude of the electric field.

**Step 7 Question 2:** Repeat the same procedure in order to calculate the electric field as a function of $r$, the distance from the axis of the cylinders for the regions $a < r < b$.

**Step 8 Question:** Make a graph in the space below of the magnitude of the electric field as a function of the parameter specifying the Gaussian surface for all regions of space.

**Penultimate Question:** What is the potential difference between $r = a$ and $r = 0$? That is, what is $\Delta V = V(a) - V(0)$?

**Final Question:** What is the potential difference between $r = b$ and $r = a$? That is, what is $\Delta V = V(b) - V(a)$?