Lecture 23: Outline

Hour 1:
- Concept Review / Overview
- PRS Questions – possible exam questions

Hour 2:
- Sample Exam

Yell if you have any questions

7:30-9 pm Tuesday
Exam 2 Topics

- DC Circuits
  - Current & Ohm’s Law (Macro- and Microscopic)
  - Power
  - Kirchhoff’s Loop Rules
  - Charging/Discharging Capacitor (RC Circuits)
- Magnetic Fields
  - Force due to Magnetic Field (Lorentz Force)
  - Magnetic Dipoles
  - Generating Magnetic Fields
    - Biot-Savart Law & Ampere’s Law
General Exam Suggestions

• You should be able to complete every problem
  • If you are confused, ask
  • If it seems too hard, you aren’t thinking enough
  • Look for hints in other problems
  • If you are doing math, you’re doing too much
• Read directions completely (before & after)
• Write down what you know before starting
• Draw pictures, define (label) variables
  • Make sure that unknowns drop out of solution
• Don’t forget units!
What You Should Study

- Review Friday Problem Solving (& Solutions)
- Review In Class Problems (& Solutions)
- Review PRS Questions (& Solutions)
- Review Problem Sets (& Solutions)
- Review PowerPoint Presentations
- Review Relevant Parts of Study Guide (& Included Examples)
Current & Ohm’s Law

Ohm’s Laws

\[ \vec{E} = \rho \vec{J} = \left( \frac{1}{\sigma} \right) \vec{J} \]

\[ \Delta V = IR \]

\[ R = \frac{\rho l}{A} \]
Series vs. Parallel

**Series**
- Current same
- Voltages add

**Parallel**
- Currents add
- Voltages same

\[
R_s = R_1 + R_2 \\
\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \\
\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \\
C_P = C_1 + C_2
\]
PRS Questions: Light Bulbs

Class 10
Current, Voltage & Power

- **Battery**
  \[ \Delta V = +\varepsilon \]

- **Resistor**
  \[ \Delta V = -IR \]

- **Capacitor**
  \[ \Delta V = -\frac{Q}{C} \]

**Supplied Power**
\[ P_{\text{supplied}} = I \Delta V = I \varepsilon \]

**Dissipated Power**
\[ P_{\text{dissipated}} = I \Delta V = I^2 R = \frac{\Delta V^2}{R} \]

**Absorbed Power**
\[ P_{\text{absorbed}} = I \Delta V = \frac{dQ}{dt} \frac{Q}{C} = \frac{d}{dt} \frac{Q^2}{2C} = \frac{dU}{dt} \]
Kirchhoff’s Rules

\[ I_1 = I_2 + I_3 \]

\[ \Delta V = -\oint \vec{E} \cdot d\vec{s} = 0 \]

Closed Path
(Dis)Charging A Capacitor

\[ I = \pm \frac{dQ}{dt} \]

\[ Q = C\varepsilon \left(1 - e^{-t/RC}\right) \]

\[ \sum_{i} \Delta V_i = \varepsilon - \frac{Q}{C} - IR = 0 \]

\[ Q_{\text{final}} - Q - RC \frac{dQ}{dt} = 0 \]

\[ I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC} \]
General Comment: RC

All Quantities Either:

\[ \text{Value}(t) = \text{Value}_{\text{Final}} \left(1 - e^{-t/\tau}\right) \]

\[ \text{Value}(t) = \text{Value}_0 e^{-t/\tau} \]

\( \tau \) can be obtained from differential equation (prefactor on \( d/dt \)) e.g. \( \tau = RC \)
PRS Questions:
DC Circuits with Capacitors

Class 12
Right Hand Rules

1. Torque: Thumb = torque, Fingers show rotation

2. Feel: Thumb = I, Fingers = B, Palm = F

3. Create: Thumb = I Fingers (curl) = B

4. Moment: Fingers (curl) = I Thumb = Moment (=B inside loop)
Magnetic Force

\[ \vec{F}_B = q \vec{v} \times \vec{B} \]

\[ d\vec{F}_B = I d\vec{s} \times \vec{B} \]

\[ \vec{F}_B = I \left( \vec{L} \times \vec{B} \right) \]
PRS Questions: Right Hand Rule

Class 14
Magnetic Dipole Moments

\[ \vec{\mu} \equiv IA \hat{n} \equiv I \vec{A} \]

Generate:

Feel:
1) Torque to align with external field
2) Forces as for bar magnets
Helmholtz Coil

Common Concept Question
Parallel (Helmholtz) makes uniform field (torque, no force)
Anti-parallel makes zero, non-uniform field (force, no torque)
PRS Questions: Magnetic Dipole Moments

Class 17
The Biot-Savart Law

Current element of length $ds$ carrying current $I$ (or equivalently charge $q$ with velocity $v$) produces a magnetic field:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{V} \times \hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, ds \times \hat{r}}{r^2}$$
Biot-Savart: 2 Problem Types

Notice that \( r \) is the same for every point on the loop. You don’t really need to integrate (except to find path length)
Ampere’s Law: \[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}} \]

- **Long Circular Symmetry**
- **(Infinite) Current Sheet**
- **Solenoid** = 2 Current Sheets
- **Torus/Coax**
PRS Questions:
Making B Fields

Classes 14-19
SAMPLE EXAM:
Problem 1: Wire Loop

A current flowing in the circuit pictured produces a magnetic field at point P pointing out of the page with magnitude B.

a) What direction is the current flowing in the circuit?

b) What is the magnitude of the current flow?
Solution 1: Wire Loop

a) The current is flowing counter-clockwise, as shown above.

b) There are three segments of the wire: the semi-circle, the two horizontal leads, and the two vertical leads. The two vertical leads do not contribute to the B field (ds || r). The two horizontal leads make an infinite wire a distance D from the field point.
Solution 1: Wire Loop

For infinite wire use Ampere’s Law:
\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}} \Rightarrow B \cdot 2\pi D = \mu_0 I \]
\[ B = \frac{\mu_0 I}{2\pi D} \]

For the semi-circle use Biot-Savart:
\[ d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{s} \times \hat{r}}{r^2} \quad r = \frac{D}{2} \text{ and } d\mathbf{s} \perp \hat{r} \]
\[ B = \int d\mathbf{B} = \int \frac{\mu_0}{4\pi} \frac{I d\mathbf{s} \times \hat{r}}{r^2} \]
\[ = \frac{\mu_0}{4\pi} \frac{I}{r^2} (\pi r) = \frac{\mu_0 I}{4r} = \frac{\mu_0 I}{2D} \]
Solution 1: Wire Loop

Adding together the two parts:

\[ B = \frac{\mu_0 I}{2\pi D} + \frac{\mu_0 I}{2D} = \frac{\mu_0 I}{2D} \left( 1 + \frac{1}{\pi} \right) \]

They gave us B and want I to make that B:

\[ I = \frac{2DB}{\mu_0 \left( 1 + \frac{1}{\pi} \right)} \]
Problem 2: RC Circuit

Initially C is uncharged.

1. When the switch is first closed, what is the current $i_3$?

2. After a very long time, how much charge is stored on the capacitor?

3. Obtain a differential equation for the charge on the capacitor
   (Here only, let $R_1=R_2=R_3=R$)

Now the switch is opened

4. Immediately after opening the switch, what is $i_1$? $i_2$? $i_3$?

5. How long before $i_2$ falls to $1/e$ of this initial value?
Solution 2: RC Circuit

Initially C is uncharged → Looks like short

\[ R_{eq} = R_3 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \implies i_3 = \frac{\varepsilon}{R_{eq}} \]
Solution 2: RC Circuit

After a long time, C is full → $i_2 = 0$

\[ i_1 = i_3 = \frac{\varepsilon}{R_1 + R_3} \]

\[ Q = C V_C = C (i_1 R_1) = C \varepsilon \frac{R_1}{R_1 + R_3} \]
Solution 2: RC Circuit

Kirchhoff’s Loop Rules

Left: \(-i_3 R + \varepsilon - i_1 R = 0\)
Right: \(-i_3 R + \varepsilon - i_2 R - \frac{q}{c} = 0\)

Current: \(i_3 = i_1 + i_2\)

Want to have \(i_2\) and \(q\) only \((L - 2R)\):

\[
0 = -(i_1 + i_2)R + \varepsilon - i_1 R + 2(i_1 + i_2)R - 2\varepsilon + 2i_2 R + \frac{2q}{c}
\]

\[
= 3i_2 R - \varepsilon + \frac{2q}{c}
\]

\[
i_2 = + \frac{dq}{dt}
\]

\[
\frac{dq}{dt} = \frac{\varepsilon}{3R} - \frac{2q}{3RC}
\]
Solution 2: RC Circuit

Now open the switch.

\[ i_3 = 0 \]

Capacitor now like a battery, with:

\[
V_C = \frac{Q}{C} = \varepsilon \frac{R_1}{R_1 + R_3}
\]

\[
i_1 = -i_2 = \frac{V_C}{R_1 + R_2} = \varepsilon \frac{R_1}{R_1 + R_3} \frac{1}{R_1 + R_2}
\]
Solution 2: RC Circuit

How long to fall to 1/e of initial current? The time constant!

This is an easy circuit since it just looks like a resistor and capacitor in series, so:

\[ \tau = (R_1 + R_2)C \]

Notice that this is different than the charging time constant, because there was another resistor in the circuit during the charging.
Problem 3: Non-Uniform Slab

Consider the slab at left with non-uniform current density:

\[ \mathbf{J} = J_0 \frac{|x|}{d} \hat{k} \]

Find B everywhere
Solution 3: Non-Uniform Slab

Direction: Up on right, down on left

Inside: (at $0<x<d$):

$$\int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc}$$

$$\int \mathbf{B} \cdot d\mathbf{s} = B\ell + 0 + 0 + 0$$

$$\mu_0 I_{enc} = \mu_0 \int \int \mathbf{J} \cdot d\mathbf{A} = \mu_0 \int_0^x \frac{J_0x}{d} \ell dx$$

$$= \mu_0 \frac{J_0\ell}{d} \frac{x^2}{2}$$

$$B = \mu_0 \frac{J_0}{d} \frac{x^2}{2} \text{ up}$$
Solution 3: Non-Uniform Slab

Direction: Up on right, down on left

Outside: \((x > d)\):

\[
\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}
\]

\[
\oint \vec{B} \cdot d\vec{s} = B\ell + 0 + 0 + 0
\]

\[
\mu_0 I_{enc} = \mu_0 \iint \vec{J} \cdot d\vec{A} = \mu_0 \int_0^d \frac{J_0x}{d} \ell dx
\]

\[
= \mu_0 \frac{J_0\ell d^2}{d \cdot 2}
\]

\[
B = \frac{1}{2} \mu_0 J_0d \text{ up}
\]
Problem 4: Solenoid

A current $I$ flows up a very long solenoid and then back down a wire lying along its axis, as pictured. The wires are negligibly small (i.e. their radius is 0) and are wrapped at $n$ turns per meter.

a) What is the force per unit length (magnitude and direction) on the straight wire due to the current in the solenoid?

b) A positive particle (mass $m$, charge $q$) is launched inside of the solenoid, at a distance $r = a$ to the right of the center. What velocity (direction and non-zero magnitude) must it have so that the field created by the wire along the axis never exerts a force on it?
Solution 4: Solenoid

SUPERPOSITION
You can just add the two fields from each part individually

a) Force on wire down axis
Since the current is anti-parallel to the field produced by the solenoid, there is no force (F=0) on this wire

b) Launching Charge q
The central wire produces a field that wraps in circles around it. To not feel a force due to this field, the particle must always move parallel to it – it must move in a circle of radius \( a \) (since that is the radius it was launched from).
b) Launching Charge $q$

So first we should use Ampere’s law to calculate the field due to the solenoid:

$$\oint \vec{B} \cdot d\vec{s} = Bl = \mu_0 NI$$

Now we just need to make a charge $q$ move in a circular orbit with $r = a$:

$$\vec{F}_B = q\vec{v} \times \vec{B} = qvB = m\frac{v^2}{r} = mv^2/a$$

$$v = \frac{qBa}{m} = \frac{q\mu_0 nIa}{m}$$

out of the page
Problem 5: Coaxial Cable

Consider a coaxial cable of with inner conductor of radius $a$ and outer conductor of inner radius $b$ and outer radius $c$. A current $I$ flows into the page on the inner conductor and out of the page on the outer conductor.

What is the magnetic field everywhere (magnitude and direction) as a function of distance $r$ from the center of the wire?
Solution 5: Coaxial Cable

Everywhere the magnetic field is clockwise. To figure out the magnitude use Ampere’s Law:

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}} \Rightarrow B \cdot 2\pi r = \mu_0 I_{\text{enc}} \]

\[ \Rightarrow B = \frac{\mu_0 I_{\text{enc}}}{2\pi r} \]

Drawn for \( a < r < b \)

The amount of current penetrating our Amperian loop depends on the radius \( r \):

\[ r \leq a: \quad I_{\text{enc}} = I \frac{r^2}{a^2} \quad \Rightarrow \quad B = \frac{\mu_0 I r}{2\pi a^2} \text{ clockwise} \]
Solution 5: Coaxial Cable

Remember: Everywhere

\[ B = \frac{\mu_0 I_{\text{enc}}}{2\pi r} \text{ clockwise} \]

\[ a \leq r \leq b: \ I_{\text{Encl}} = I \implies B = \frac{\mu_0 I}{2\pi r} \text{ clockwise} \]

\[ b \leq r \leq c: \ I_{\text{Encl}} = I \left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) \]

\[ \implies B = \frac{\mu_0 I}{2\pi r} \left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) \text{ clockwise} \]
Solution 5: Coaxial Cable

Remember: Everywhere

\[ B = \frac{\mu_0 I_{enc}}{2\pi r} \text{ clockwise} \]

\[ r \geq c: \quad I_{Encl} = 0 \quad \Rightarrow \quad B = 0 \]