Class 24: Outline

Hour 1:
   Inductance & LR Circuits

Hour 2:
   Energy in Inductors
Last Time:
Faraday’s Law
Mutual Inductance
Faraday’s Law of Induction

\[ \mathcal{E} = -N \frac{d\Phi}{dt} \]

Changing magnetic flux *induces* an EMF

Lenz: Induction *opposes* change
A current $I_2$ in coil 2, induces some magnetic flux $\Phi_{12}$ in coil 1. We define the flux in terms of a “mutual inductance” $M_{12}$:

$$N_1 \Phi_{12} \equiv M_{12} I_2$$

$$\rightarrow M_{12} = \frac{N_1 \Phi_{12}}{I_2}$$

$$\mathcal{E}_{12} \equiv -M \frac{dI_2}{dt}$$

You need AC currents!
Demonstration: Remote Speaker
This Time: Self Inductance
Self Inductance

What if we forget about coil 2 and ask about putting current into coil 1? There is “self flux”:

\[ N_1 \Phi_{11} \equiv M_{11} I_1 \equiv LI \]

\[ L = \frac{N\Phi}{I} \]

\[ \mathcal{E} \equiv -L \frac{dI}{dt} \]
Calculating Self Inductance

\[ L = \frac{N\Phi}{I} \]

Unit: Henry
\[ 1 \text{ H} = 1 \frac{V \cdot \text{s}}{\text{A}} \]

1. Assume a current \( I \) is flowing in your device
2. Calculate the B field due to that \( I \)
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out \( I \))
Group Problem: Solenoid

Calculate the self-inductance $L$ of a solenoid (n turns per meter, length $\ell$, radius $R$)

**REMEMBER**
1. Assume a current $I$ is flowing in your device
2. Calculate the $B$ field due to that $I$
3. Calculate the flux due to that $B$ field
4. Calculate the self inductance (divide out $I$)

$$L = N\Phi/I$$
Inductor Behavior

\[ \mathcal{E} = -L \frac{dI}{dt} \]

Inductor with constant current does nothing
Back EMF $\varepsilon = -L \frac{dI}{dt}$

- Lenz's law emf

$\frac{dI}{dt} > 0$, $\varepsilon_L < 0$

$\frac{dI}{dt} < 0$, $\varepsilon_L > 0$
Inductors in Circuits

Inductor: Circuit element which exhibits self-inductance

Symbol: 

When traveling in direction of current:

\[ \mathcal{E} = -L \frac{dI}{dt} \]

Inductors hate change, like steady state
They are the opposite of capacitors!
PRS Question:
Closing a Switch
\[ \sum V_i = \varepsilon - IR - L \frac{dI}{dt} = 0 \]
LR Circuit

\[ \mathcal{E} - IR - L \frac{dI}{dt} = 0 \quad \Rightarrow \quad \frac{L}{R} \frac{dI}{dt} = -\left( I - \frac{\mathcal{E}}{R} \right) \]

Solution to this equation when switch is closed at \( t = 0 \):

\[ I(t) = \frac{\mathcal{E}}{R} \left( 1 - e^{\frac{-t}{\tau}} \right) \]

\[ \tau = \frac{L}{R} : \text{LR time constant} \]
t=0⁺: Current is trying to change. Inductor works as hard as it needs to to stop it

t=∞: Current is steady. Inductor does nothing.
LR Circuit

t=0⁺: Current is trying to change. Inductor works as hard as it needs to stop it.
t=∞: Current is steady. Inductor does nothing.
General Comment: LR/RC

All Quantities Either:

\[ \tau = \frac{L}{R} \quad \text{or} \quad \tau = RC \]

\[ \text{Value}(t) = \text{Value}_{\text{Final}} \left(1 - e^{-t/\tau}\right) \]

\[ \text{Value}(t) = \text{Value}_{0} e^{-t/\tau} \]

\(\tau\) can be obtained from differential equation
(prefactor on d/dt) e.g. \(\tau = L/R\) or \(\tau = RC\)
1. What direction does the current flow just after turning off the battery (at $t=0+$)? At $t=\infty$?

2. Write a differential equation for the circuit.

3. Solve and plot $I$ vs. $t$ and voltmeters vs. $t$. 

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**Group Problem: LR Circuit**
PRS Questions:
LR Circuit & Problem...
Non-Conservative Fields

\[ \int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]

E is no longer a conservative field – Potential now meaningless
This concept (& next 3 slides) are complicated. Bare with me and try not to get confused
Kirchhoff’s Modified 2nd Rule

\[
\sum_i \Delta V_i = - \oint \mathbf{E} \cdot d\mathbf{s} = +N \frac{d\Phi_B}{dt}
\]

\[
\Rightarrow \sum_i \Delta V_i - N \frac{d\Phi_B}{dt} = 0
\]

If all inductance is ‘localized’ in inductors then our problems go away – we just have:

\[
\sum_i \Delta V_i - L \frac{dI}{dt} = 0
\]
Ideal Inductor

BUT, EMF generated in an inductor is not a voltage drop across the inductor!

\[ \varepsilon = -L \frac{dI}{dt} \]

\[ \Delta V_{\text{inductor}} \equiv -\int \vec{E} \cdot d\vec{s} = 0 \]

Because resistance is 0, E must be 0!
Conclusion:
Be mindful of physics
Don’t think too hard doing it
Demos:
Breaking circuits with inductors
Internal Combustion Engine

See figure 1:
http://auto.howstuffworks.com/engine3.htm
The Distributor:
http://auto.howstuffworks.com/ignition-system4.htm

(A) High Voltage Lead
(B) Cap/Rotor Contact
(C) Distributor Cap
(D) To Spark Plug

(A) Coil connection
(B) Breaker Points
(D) Cam Follower
(E) Distributor Cam
Modern Ignition

See figure:
http://auto.howstuffworks.com/ignition-system.htm
Energy in Inductor
Energy Stored in Inductor

\[ \mathcal{E} = +IR + L \frac{dI}{dt} \]

\[ I \mathcal{E} = I^2 R + L I \frac{dI}{dt} \]

Battery Supplies
Resistor Dissipates
Inductor Stores
Energy Stored in Inductor

\[ U_L = \frac{1}{2} L I^2 \]

But where is energy stored?
Example: Solenoid

Ideal solenoid, length $l$, radius $R$, $n$ turns/length, current $I$:

$$B = \mu_0 n I$$
$$L = \mu_0 n^2 \pi R^2 l$$

$$U_B = \frac{1}{2} LI^2 = \frac{1}{2} \left( \mu_0 n^2 \pi R^2 l \right) I^2$$

$$U_B = \left( \frac{B^2}{2\mu_0} \right) \pi R^2 l$$

Energy Density

Volume
Energy Density

Energy is stored in the magnetic field!

\[ u_B = \frac{B^2}{2\mu_0} \quad : \text{Magnetic Energy Density} \]

\[ u_E = \frac{\varepsilon_0 E^2}{2} \quad : \text{Electric Energy Density} \]
Group Problem: Coaxial Cable

1. How much energy is stored per unit length?
2. What is inductance per unit length?

HINTS: This does require an integral
The EASIEST way to do (2) is to use (1)
Back to Back EMF
PRS Question: Stopping a Motor