Class 25: Outline

Hour 1:
Expt. 10: Part I: Measuring L
LC Circuits

Hour 2:
Expt. 10: Part II: LRC Circuit
Last Time:
Self Inductance
Self Inductance

To Calculate: \( L = \frac{N \Phi}{I} \)

1. Assume a current \( I \) is flowing in your device
2. Calculate the B field due to that \( I \)
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out \( I \))

The Effect: Back EMF: \( \mathcal{E} \equiv -L \frac{dI}{dt} \)

Inductors hate change, like steady state
They are the opposite of capacitors
**LR Circuit**

$t=0^+$: Current is trying to change. Inductor works as hard as it needs to to stop it.

$t=\infty$: Current is steady. Inductor does nothing.
LR Circuit: AC Output Voltage

- **Output Voltage**
- **Voltmeter across L**
- **Current**

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Output (V)</th>
<th>Inductor (V)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
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<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
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<td>0.00</td>
<td>-1.00</td>
<td>-0.50</td>
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<tr>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
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</table>
Non-Ideal Inductors

Non-Ideal (Real) Inductor: Not only L but also some R

In direction of current: \( \mathcal{E} = -L \frac{dI}{dt} - IR \)
LR Circuit w/ Real Inductor

1. Time constant from I or V
2. Check inductor resistance from V just before switch

Due to Resistance
Experiment 10:
Part I: Measure L, R

STOP
after you do Part I of Experiment 10 (through page E10-5)
LC Circuits
Mass on a Spring:
Simple Harmonic Motion
(Demonstration)
Mass on a Spring

What is Motion?

\[ F = -kx = ma = m \frac{d^2x}{dt^2} \]

\[ m \frac{d^2x}{dt^2} + kx = 0 \]

Simple Harmonic Motion

\[ x(t) = x_0 \cos(\omega_0 t + \phi) \]

\[ \omega_0 = \sqrt{\frac{k}{m}} = \text{Angular frequency} \]

- \( x_0 \): Amplitude of Motion
- \( \phi \): Phase (time offset)
Mass on a Spring: Energy

\( x(t) = x_0 \cos(\omega_0 t + \phi) \quad \quad x'(t) = -\omega_0 x_0 \sin(\omega_0 t + \phi) \)

Energy has 2 parts: (Mass) Kinetic and (Spring) Potential

\[
K = \frac{1}{2} m \left( \frac{dx}{dt} \right)^2 = \frac{1}{2} k x_0^2 \sin^2(\omega_0 t + \phi)
\]

\[
U_s = \frac{1}{2} k x^2 = \frac{1}{2} k x_0^2 \cos^2(\omega_0 t + \phi)
\]

Energy sloshes back and forth
Simple Harmonic Motion

Amplitude \((x_0)\)

Period \((T)\) = \(\frac{1}{\text{frequency} \,(f)}\)

\[= \frac{2\pi}{\text{angular frequency} \,(\omega)}\]

Phase Shift \((\phi)\) = \(\frac{\pi}{2}\)

\[x(t) = x_0 \cos(\omega_0 t - \phi)\]
Electronic Analog: LC Circuits
Analog: LC Circuit

Mass doesn’t like to accelerate

Kinetic energy associated with motion

\[ F = ma = m \frac{dv}{dt} = m \frac{d^2x}{dt^2}; \quad E = \frac{1}{2} m v^2 \]

Inductor doesn’t like to have current change

Energy associated with current

\[ \varepsilon = -L \frac{dI}{dt} = -L \frac{d^2q}{dt^2}; \quad E = \frac{1}{2} LI^2 \]
Analog: LC Circuit

Spring doesn’t like to be compressed/extended
Potential energy associated with compression

\[ F = -kx; \quad E = \frac{1}{2}kx^2 \]

Capacitor doesn’t like to be charged (+ or -)
Energy associated with stored charge

\[ \varepsilon = \frac{1}{C}q; \quad E = \frac{1}{2} \frac{1}{C}q^2 \]

\[ F \rightarrow \varepsilon; \quad x \rightarrow q; \quad v \rightarrow I; \quad m \rightarrow L; \quad k \rightarrow C^{-1} \]
1. Set up the circuit above with capacitor, inductor, resistor, and battery.
2. Let the capacitor become fully charged.
3. Throw the switch from a to b
4. What happens?
LC Circuit

It undergoes simple harmonic motion, just like a mass on a spring, with trade-off between charge on capacitor (Spring) and current in inductor (Mass)
PRS Questions:
LC Circuit
LC Circuit

\[ \frac{Q}{C} - L \frac{dI}{dt} = 0 \; ; \; I = -\frac{dQ}{dt} \]

\[ \frac{d^2Q}{dt^2} + \frac{1}{LC} Q = 0 \]

**Simple Harmonic Motion**

\[ Q(t) = Q_0 \cos(\omega_0 t + \phi) \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

- \( Q_0 \): Amplitude of Charge Oscillation
- \( \phi \): Phase (time offset)
\[ U_E = \frac{Q^2}{2C} = \left( \frac{Q_0^2}{2C} \right) \cos^2 \omega_0 t \]

\[ U_B = \frac{1}{2} LI^2 = \frac{1}{2} LI_0^2 \sin^2 \omega_0 t = \left( \frac{Q_0^2}{2C} \right) \sin^2 \omega_0 t \]

\[ U = U_E + U_B = \frac{Q^2}{2C} + \frac{1}{2} LI^2 = \frac{Q_0^2}{2C} \]

Total energy is conserved!!
Adding Damping: RLC Circuits
Damped LC Oscillations

Resistor dissipates energy and system rings down over time

Also, frequency decreases:

\[ \omega' = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2} \]
Experiment 10:
Part II: RLC Circuit

Use Units
PRS Questions:
2 Lab Questions