Class 30: Outline

Hour 1:
Traveling & Standing Waves

Hour 2:
Electromagnetic (EM) Waves
Last Time:
Traveling Waves
Traveling Sine Wave

Now consider \( f(x) = y = y_0 \sin(kx) \):

Amplitude \( (y_0) \)

Wavelength \( (\lambda) = \frac{2\pi}{\text{wavenumber} (k)} \)

What is \( g(x,t) = f(x+vt) \)? Travels to left at velocity \( v \)

\[
y = y_0 \sin(k(x+vt)) = y_0 \sin(kx+kvt)
\]
Traveling Sine Wave

\[ y = y_0 \sin(kx + kvt) \]

At \( x=0 \), just a function of time: \( y = y_0 \sin(kvt) \equiv y_0 \sin(\omega t) \)

Amplitude \( (y_0) \)

Period \( (T) = \frac{1}{\text{frequency } (f)} \)

\[ = \frac{2\pi}{\text{angular frequency } (\omega)} \]
Traveling Sine Wave

- Wavelength: $\lambda$
- Frequency: $f$
- Wave Number: $k = \frac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- Speed of Propagation: $v = \frac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

\[
y = y_0 \sin(kx - \omega t)
\]
This Time: Standing Waves
Standing Waves

What happens if two waves headed in opposite directions are allowed to interfere?

\[ E_1 = E_0 \sin(kx - \omega t) \quad E_2 = E_0 \sin(kx + \omega t) \]

Superposition: \[ E = E_1 + E_2 = 2E_0 \sin(kx) \cos(\omega t) \]
Standing Waves: Who Cares?

Most commonly seen in resonating systems:
Musical Instruments, Microwave Ovens

\[ E = 2E_0 \sin(kx) \cos(\omega t) \]
Standing Waves: Bridge

Tacoma Narrows Bridge Oscillation:
http://www.pbs.org/wgbh/nova/bridge/tacoma3.html
Group Work: Standing Waves

Do Problem 2

\[ E_1 = E_0 \sin(kx - \omega t) \quad E_2 = E_0 \sin(kx + \omega t) \]

Superposition: \[ E = E_1 + E_2 = 2E_0 \sin(kx) \cos(\omega t) \]
Last Time:
Maxwell’s Equations
Maxwell’s Equations

\[ \iint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \quad \text{(Gauss's Law)} \]

\[ \oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's Law)} \]

\[ \iiint_S \vec{B} \cdot d\vec{A} = 0 \quad \text{(Magnetic Gauss's Law)} \]

\[ \oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{(Ampere-Maxwell Law)} \]

\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad \text{(Lorentz force Law)} \]
Which Leads To…
EM Waves
Electromagnetic Radiation: Plane Waves

Traveling E & B Waves

- Wavelength: $\lambda$
- Frequency: $f$
- Wave Number: $k = \frac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- Speed of Propagation: $v = \frac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

$$\vec{E} = \hat{E} E_0 \sin(kx - \omega t)$$
Properties of EM Waves

Travel (through vacuum) with speed of light
\[ v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s} \]

At every point in the wave and any instant of time, E and B are in phase with one another, with
\[ \frac{E}{B} = \frac{E_0}{B_0} = c \]

E and B fields perpendicular to one another, and to the direction of propagation (they are transverse):
\[ \text{Direction of propagation} = \text{Direction of } \vec{E} \times \vec{B} \]
PRS Questions: Direction of Propagation
How Do Maxwell’s Equations Lead to EM Waves?

Derive Wave Equation
Wave Equation

Start with Ampere-Maxwell Eq: \[ \oint_c \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \mathbf{E} \cdot d\mathbf{A} \]
Wave Equation

Start with Ampere-Maxwell Eq: \( \oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A} \)

Apply it to red rectangle:
\[ \oint \vec{B} \cdot d\vec{s} = B_z(x,t)l - B_z(x + dx, t)l \]
\[ \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A} = \mu_0 \epsilon_0 \left( l \frac{dx}{l} \frac{\partial E_y}{\partial t} \right) \]

\[ -\frac{B_z(x + dx, t) - B_z(x, t)}{dx} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t} \]

So in the limit that \( dx \) is very small:
\[ -\frac{\partial B_z}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t} \]
Wave Equation

Now go to Faraday’s Law: \[ \oint_C \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} \]
Wave Equation

Faraday’s Law:

$$\oint_c \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$

Apply it to red rectangle:

$$\oint_c \mathbf{E} \cdot d\mathbf{s} = E_y(x + dx, t)l - E_y(x, t)l$$

$$-\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} = -ldx \frac{\partial B_z}{\partial t}$$

$$\frac{E_y(x + dx, t) - E_y(x, t)}{dx} = -\frac{\partial B_z}{\partial t}$$

So in the limit that $dx$ is very small:

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$
1D Wave Equation for $E$

\[
\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad -\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}
\]

Take $x$-derivative of 1st and use the 2nd equation

\[
\frac{\partial}{\partial x} \left( \frac{\partial E_y}{\partial x} \right) = \frac{\partial^2 E_y}{\partial x^2} = \frac{\partial}{\partial x} \left( -\frac{\partial B_z}{\partial t} \right) = -\frac{\partial}{\partial t} \left( \frac{\partial B_z}{\partial x} \right) = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}
\]

\[
\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}
\]
1D Wave Equation for E

\[
\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}
\]

This is an equation for a wave. Let: \( E_y = f (x - vt) \)

\[
\frac{\partial^2 E_y}{\partial x^2} = f''(x - vt)
\]

\[
\frac{\partial^2 E_y}{\partial t^2} = \nu^2 f''(x - vt)
\]

\[
\nu^2 = \frac{1}{\mu_0 \varepsilon_0}
\]
1D Wave Equation for B

\[ \frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \quad \frac{\partial B_z}{\partial x} = -\mu_0\varepsilon_0 \frac{\partial E_y}{\partial t} \]

Take x-derivative of 1st and use the 2nd equation

\[ \frac{\partial}{\partial t} \left( \frac{\partial B_z}{\partial t} \right) = \frac{\partial^2 B_z}{\partial t^2} = \frac{\partial}{\partial t} \left( -\frac{\partial E_y}{\partial x} \right) = -\frac{\partial}{\partial x} \left( \frac{\partial E_y}{\partial t} \right) = \frac{1}{\mu_0\varepsilon_0} \frac{\partial^2 B_z}{\partial x^2} \]

\[ \frac{\partial^2 B_z}{\partial x^2} = \mu_0\varepsilon_0 \frac{\partial^2 B_z}{\partial t^2} \]
Electromagnetic Radiation

Both E & B travel like waves:

\[
\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}
\]

But there are strict relations between them:

\[
\frac{\partial B_z}{\partial t} = - \frac{\partial E_y}{\partial x} \quad \frac{\partial B_z}{\partial x} = - \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}
\]

Here, \(E_y\) and \(B_z\) are “the same,” traveling along x axis.
Amplitudes of E & B

Let \( E_y = E_0 f(x - vt) \); \( B_z = B_0 f(x - vt) \)

\[
\frac{\partial B_z}{\partial t} = - \frac{\partial E_y}{\partial x} \implies -vB_0 f'(x - vt) = -E_0 f'(x - vt)
\]

\[
\implies vB_0 = E_0
\]

\( E_y \) and \( B_z \) are “the same,” just different amplitudes
Group Problem: EM Standing Waves

Consider EM Wave approaching a perfect conductor:

$$\mathbf{E}_{\text{incident}} = \hat{x}E_0 \cos(kz - \omega t)$$

If the conductor fills the XY plane at Z=0 then the wave will reflect and add to the incident wave

1. What must the total E field ($E_{\text{inc}} + E_{\text{ref}}$) at Z=0 be?
2. What is $E_{\text{reflected}}$ for this to be the case?
3. What are the accompanying B fields? ($B_{\text{inc}}$ & $B_{\text{ref}}$)
4. What are $E_{\text{total}}$ and $B_{\text{total}}$? What is $B(Z=0)$?
5. What current must exist at Z=0 to reflect the wave? Give magnitude and direction.

Recall: $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
Next Time: How Do We Generate Plane Waves?