8.02X Electricity and Magnetism

Practice-Quiz #4b

Solutions
Problem 1 (25 points)

In the HVPS experiment, you built a “transformer” by winding 6 loops of wire around a tightly wound red coil.

(a) Which side of the transformer was the primary side in this setup?

The 6 Loops

(b) Assume that in your setup the inner (red) coil had length L_1, number of windings N_1 and radius R_1. The outer coil (wire loops) had length L_2, number of windings N_2 and radius R_2. Derive an expression for the mutual inductance of the two coils. Show work!

(c) Based on the known output voltages of LVPS and HVPS, estimate (within a factor of 2) a numerical value for the number of windings of the red coil (ignore the different length for primary and secondary coil).

(d) Assume a current I_2(t) = I_0 \cdot \cos(\omega t) was flowing through the outer coil. What would the voltage across the red coil $\Delta V_1(t)$ be?

\[ EMT_{Coil} = -M \frac{dI_1}{dt} \]

\[ = -N_2 \cdot \pi R_1^2 \cdot \frac{dB_1}{dt} \]

\[ = -N_2 \cdot \pi R_1^2 \cdot \frac{M}{L_1} \cdot \frac{dI}{dt} \]

\[ \frac{d\Phi_{B_2}}{dt} \]

\[ \Rightarrow M = \frac{N_2 \cdot \pi R_1^2 \cdot M_0}{L_1} \]

\[ V_{LVPS} \approx 10 V \]

\[ V_{HVPS} \approx 1000 V \]

\[ \frac{V_{HVPS}}{V_{LVPS}} = \frac{N_{HVPS}}{N_{LVPS}} \Rightarrow N_{HVPS} \approx 6000 \]
\[ v_1(t) = -M \frac{dI_2}{dt} \quad i(t) = I_0 \cos(\omega t) \]

\[ = -M \cdot (-I_0 \cdot \omega \cdot \sin(\omega t)) \]

\[ = M \cdot I_0 \omega \sin(\omega t) \]

with \( M = -N_2 \cdot \pi R_1^2 \cdot \frac{N_1 \cdot \mu_0}{L_1} \)
Problem 2 (25 points)

Shown below is a circuit that is connected to a DC power supply with an output voltage $V_0$. For times $t < 0$, the switch is in position 1 and a current is flowing through the inductor (inductance $L$), the resistor (resistance $R$) and the power supply. Assume the switch has been closed for a very long time and the resistance of the inductor is negligible. Assume also that for $t < 0$, the capacitor (Capacity $C$) is discharged ($Q=0$).

At $t=0$, the switch is moved to position 2 and the power supply and resistor are therefore removed from the circuit.

(a) At $t=0$, what is the total energy stored in the circuit formed by capacitor and inductor?

$$U = \frac{1}{2} L I^2 + \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} L \left( \frac{V_0}{R} \right)^2$$

(b) Give an example (sketch) of a mechanical system that corresponds to the circuit formed by the inductor and capacitor (after $t=0$). Identify which elements in the mechanical system correspond to which circuit elements.

- Mass on a spring: Inductor
- Restoring Force: Spring
- Mass: Capacitor

(c) How will the charge $Q(t)$ on the capacitor vary with time? Give an equation in terms of the quantities defined above.

$$Q(t) = Q_0 \cdot \sin(\omega t) \quad \text{with} \quad \omega = \frac{1}{\sqrt{LC}} \quad \text{and} \quad Q_0 = \sqrt{LC} \cdot I$$

(d) On the graph below, sketch how the energy in the inductor varies with time after $t=0$. 
Problem 3 (25 points)

Consider a plane wave with an amplitude that is described by the following equations:

\[ A_x = 0 \]
\[ A_y = 0 \]
\[ A_z = A_0 \cos(\omega t - (2\pi/3m) x) \]

(a) Which direction is the wave traveling in?

(b) How big is the wavelength of the wave?

(c) Could these equations describe a sound wave? Explain your answer.

(d) If the wave was electromagnetic, what would the frequency \( f \) be?

\[ \text{a/ } \text{Direction} \]
\[ \text{b/ } \lambda = 3 \text{m} \]
\[ \text{c/ } \text{No. A is } \perp \text{ to } \vec{v} \rightarrow \text{ transverse wave} \]
\[ \text{d/ } \lambda \cdot f = 3 \cdot 10^8 \frac{m}{s} \Rightarrow f = 10^8 \text{Hz} \]
Problem 4 (25 points) AMP experiment

(a) What is the purpose of the AMP experiment? (1-2 sentences)

(b) How did you calibrate the AMP setup? What does the calibration curve tell you? (2-3 sentences)

(c) Shown below is a calibration circuit like that on the AMP experiment. All voltages are measured relative to the common line C, which is defined as 0V. What is the voltage at point X when the slider of the potentiometer is 1/2 way between the extreme positions?

(d) What are the maximum and minimum voltages at point D relative to C, when the slider is moved from one extreme position to the other?
a) Purpose: To "amplify" the input voltage, i.e. create an output signal that is proportional to the input, but larger a factor "g", the gain.

b) First shorted the input and zeroed the output using 100 kΩ pot, with HMM on the 250 mV setting. Then connected calibration output D to input A. Vary input voltage from -67 mV to 67 mV and record output voltage as a function of input voltage.

c) By symmetry $V(x) = 0$

d) Equivalent diagram

\[
\begin{align*}
A & \quad 0 \text{mV} \\
D & \quad R_1 = \left( \frac{1}{1.3kΩ} + \frac{1}{5.5kΩ} \right)^{-1} = 1kΩ \\
R_2 & = 91kΩ \\
B & \quad +6V \text{ or } -6V
\end{align*}
\]

\[
V_B = \frac{R_1}{R_1 + R_2} \cdot V_{AB} = \frac{1}{92} \cdot 6V \quad \text{(slider left)}
\]

\[
= \frac{1}{92} \cdot 6V \quad \text{(slider right)}
\]