Electricity and Magnetism

• Last time: Electric Field
• Today:
  - Electric Flux
  - Gauss’ Law
Gauss’ Law

• Today: Gauss’ Law
  – Not so many demos, some math

• Electricity and Magnetism in 4 equations:

\[
\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0} \\
\oint \mathbf{B} \cdot d\mathbf{A} = 0 \\
\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \\
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}
\]
Electric Flux

• Definition (simple case):

$$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

• What does that mean?
  – Analogy with flow of e.g. water
‘Flux’ of water

Flow: \( \frac{dV}{dt} = A \frac{dx}{dt} = Av \)
`Flux` of water

Velocity $v$

Flow: $A v \cos(\alpha) = \vec{A} \cdot \vec{v}$
Electric Flux

• Electric Flux: $\Phi_E = \mathbf{E} \cdot \mathbf{A}$

• Same mathematical form as water flow

• But there is no ‘substance’ flowing

• Took almost a century to accept

• Flux tells us how much field ‘passes’ through surface A

Feb 19 2002
Electric Flux

- For ‘complicated’ surfaces:
  - Use integral

\[ \Phi_E = \int_A \vec{E} \cdot d\vec{A} \]

- Often, ‘closed’ surfaces

\[ \Phi_E = \oint_A \vec{E} \cdot d\vec{A} \]
Electric Flux

- Example of closed surface: Box

- Flux in (left) = -Flux out (right): $\Phi_E = 0$

- No ‘source’ of flux in this box
Electric Flux

• How to make $\Phi_E$ non-zero?
• Remember:
  - Put Charge Q inside ‘box’!

Feb 19 2002
Gauss’ Law

• How are flux and charge connected?

• Charge $Q_{\text{encl}}$ as source of flux through closed surface

$$\oint_{A} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$$
Gauss’ Law

• Gauss vs Coulomb

\[
E = k \frac{Q}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \text{ with } \varepsilon_0 = \frac{1}{4\pi k}
\]
Gauss’ Law

- Gauss vs Coulomb

$$\frac{Q_{encl}}{\epsilon_0} = \oint_{\text{sphere}} \vec{E} \cdot d\vec{A} =$$

$$\oint_{\text{sphere}} EdA =$$

$$E \oint_{\text{sphere}} dA =$$

$$E(4\pi r^2) \implies$$

$$E = \frac{1}{4\pi \epsilon_0} \frac{Q_{encl}}{r^2}$$
Example

- Solid charged sphere (non-conducting)

$A_1$  

\[
(1) \ r > r_0 : \\
\oint_{A_1} \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0} \\
\implies E = \frac{1}{4\pi \varepsilon_0 r^2} Q
\]
Example

• Solid charged sphere (non-conducting)

\[ (2) \ r > r_0 : \]
\[ \oint_{A_2} \vec{E} \cdot d\vec{A} = \]
\[ E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0} = \frac{4}{3} \pi r^3 Q = \frac{r^3}{r_0^3} Q \]

\[ E(4\pi r^2) = \frac{r^3}{r_0^3} Q \implies E = \frac{1}{4\pi \epsilon_0 r_0^3} Q r \]
Example

• Solid charged sphere (non-conducting)
Example II

- Conducting Sphere

\[ E = 0 \]

\[ E \sim \frac{1}{r^2} \]
Example IIa

- Conducting Sphere
Example III

- Line of Charge

\[ \lambda: \text{Charge density } \frac{dQ}{dl} \]

\[ \int_{cyl} \vec{E} \cdot d\vec{A} = E(2\pi rl) = \frac{Q_{encl}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0} \]

\[ \Rightarrow E = \frac{1}{2\pi \epsilon_0} \frac{\lambda}{r} \]