Electricity and Magnetism

• Capacitors
  – Dielectric

• Experiment EF
Parallel Plate Capacitor

\[ C = \varepsilon_0 \frac{A}{d} \]

- Change \( d \)
  - change \( C \)
- \( Q \) constant

\[ \text{d bigger} \rightarrow C \text{ smaller} \rightarrow V \text{ bigger for fixed } Q \]

\[ \text{slope} = \frac{1}{C} \]
Energy stored in Capacitor

- Can store more energy, if
  - $C$ bigger
  - $V$ bigger

$$W_{tot} = \frac{1}{2}CV^2$$
In-Class Demo

$C = 100 \mu F$

$V_{ab} = 4000 \text{ V}$

$U = 800 \text{ J}$

thin wire
Where is the energy stored?

- Energy is stored in Electric Field

\[ U_{\text{stored}} = \frac{1}{2}CV^2 = \frac{1}{2}(\varepsilon_0 \frac{A}{d})(E \cdot d)^2 \]

\[ = \frac{1}{2} \varepsilon_0 E^2 \text{ Volume} \]

- \( E^2 \) gives Energy Density:
- \( U/ \text{ Volume} = \frac{1}{2} \varepsilon_0 E^2 \)

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Electric Circuits

Capacitor

Wire

Voltage source (like LVPS)

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Electric Circuits

- Two capacitors in parallel
- $V_{56} = V_{23} = V_{14}$ (after capacitor is charged)
- $Q_1/C_1 = Q_2/C_2 = V_{14}$
- $Q_{tot} = Q_1 + Q_2$
- $C_{tot} = (Q_1 + Q_2)/ V_{14} = C_1 + C_2$

- Capacitors in parallel -> Capacitances add!
Electric Circuits

- Two capacitors in **series**
- $V_{14} = V_{23} + V_{56}$
- $Q = Q_1 = Q_2$
- $V_{tot} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \frac{Q}{C_1 + C_2}$
- $\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2}$
- Inverse Capacitances add!
Dielectrics

- Parallel Plate Capacitor:
  - \( C = \varepsilon_0 \frac{A}{d} \)
  - Ex. \( A = 1\,\text{m}^2, \, d=0.1\,\text{mm} \)
    \[ \Rightarrow C \sim 0.1\,\mu\text{F} \]

- How do they do that?
- Where to get a factor of 10000?
Dielectric Demo

- Start w/ charged capacitor
- $d$ big $\rightarrow$ $C$ small $\rightarrow$ $V$ large
Dielectric Demo

- Start w/ charged capacitor
- $d$ big -> $C$ small -> $V$ large
- Insert Glass plate
- Now $V$ much smaller
- $C$ bigger
- But $A$ and $d$ unchanged!
- Glass is a **Dielectric**
Microscopic view

Remember: **Dipoles**

\[
\vec{p} = q \, \vec{L}
\]

\[\vec{E} = 0\]

\[\vec{E} > 0\]
Microscopic view

Def: **Polarization** \( \mathbf{P} = n \langle \mathbf{p} \rangle = \text{const.} \mathbf{E} \)

Density: \#dipoles/volume

\( \mathbf{E} = 0 \) vs. \( \mathbf{E} > 0 \)
Microscopic view

Polarization $\vec{P} = \text{const.}$  $\vec{E} = \varepsilon_0 \chi \vec{E}$

Glass

+Q

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-Q
Microscopic view

Polarization  \( \vec{P} = \text{const.} \)  \( \vec{E} = \varepsilon_0 \chi \vec{E} \)
Microscopic view

Inside: Charges compensate

Surface: Unbalanced Charges!

+Q

- Q

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Microscopic view

Inside: Charges compensate

Surface: Unbalanced Charges!

Surface charges reduce field!
Dielectric Constant

\[ |\sigma_p^+| = |\sigma_p^-| = \frac{|Q_p|}{A} \]

Surface charge density

\[ = n q \frac{L A}{A} \]

\[ = n p \]

\[ = P \]  Polarization
Dielectric Constant

\[ E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} - \frac{|\sigma_p^+|}{2\varepsilon_0} - \frac{|\sigma_p^-|}{2\varepsilon_0} \]

Add contributions to \( E \)

\[ = \frac{\sigma}{\varepsilon_0} - \frac{P}{\varepsilon_0} \]

\[ = E_0 + \chi E \]

\[ \Rightarrow E = \frac{E}{1 + \chi} \equiv \frac{E_0}{K} \]

Field w/o Dielectric

K: Dielectric Constant

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Dielectric Constant

- Dielectric reduces field $E_0$ ($K > 1$)
  - $E = \frac{1}{K} E_0$
- Dielectric increases Capacitance
  - $C = \frac{Q}{V} = \frac{Q}{(E \cdot d)} = K \frac{Q}{(E_0 \cdot d)}$
- This is how to make small capacitors with large $C$!
## Dielectric Constant

### Examples

<table>
<thead>
<tr>
<th>Material</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1</td>
</tr>
<tr>
<td>Air</td>
<td>1.0006</td>
</tr>
<tr>
<td>Plexiglass</td>
<td>3.4</td>
</tr>
<tr>
<td>Water</td>
<td>80.4</td>
</tr>
<tr>
<td>Ethanol</td>
<td>23</td>
</tr>
<tr>
<td>Ceramics</td>
<td>~5000</td>
</tr>
<tr>
<td>Glass</td>
<td>5-10</td>
</tr>
</tbody>
</table>

Similar to vacuum

Large!

C in HVPS

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‘Puzzle’ Demo

Copper

Glass

\[ \text{Diagram of a circuit with a copper and glass element, connected to a battery.} \]
‘Puzzle’ Demo
‘Puzzle’ Demo

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‘Puzzle’ Demo
‘Puzzle’ Demo

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‘Puzzle’ Demo

Surfaces charges remain on Glass!
‘Puzzle’ Demo
‘Puzzle’ Demo
Experiment EF

How do we measure $\varepsilon_0$ with this?
\[ F_{onfoil} = Q_{foil} \vec{E}_{top} \]

\[ \vec{E}_{top} = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0} \frac{1}{A_{wash}} (-\hat{y}) \]

\[ Q_{foil} = -Q \frac{A_{foil}}{A_{wash}} \]

\[ \Rightarrow \vec{F}_{onfoil} = -Q \frac{A_{foil}}{A_{wash}} \frac{Q}{A_{wash}2\varepsilon_0} (-\hat{y}) \]

\[ Q = CV = \varepsilon_0 A/d \ V \]

\[ \vec{F}_{onfoil} = \frac{(\varepsilon_0 V A_{wash}/d)^2}{A_{wash}^2} \frac{A_{foil}}{2\varepsilon_0} \hat{y} \]

\[ = \frac{\varepsilon_0 V^2}{2d^2} A_{foil} \hat{y} \]
How to get force?

\[ F_{\text{foil}} = m_\text{g} \quad \text{with} \quad m_N = \rho_m A_{\text{foil}} N t \]

\[ \Rightarrow \frac{\epsilon_0 V^2}{2d^2} A_{\text{foil}} = \rho_m A_{\text{foil}} N t g \]

\[ \Rightarrow \frac{\epsilon_0 V^2}{2d^2} = \rho_m N t g \]

\[ \Rightarrow V^2 = \frac{2d^2 \rho_m t g}{\epsilon_0} N \]

\[ = \text{slope} \cdot N \]

\[ \Rightarrow \epsilon_0 = \frac{\rho_m 2d^2 t g}{\text{slope}} \]

Balance unknown Force with known Force -> Gravity!