Electricity and Magnetism

• Today
  – DC Circuits
  – Kirchoff’s Rules
  – RC Circuits
Ohm’s law

• Def. $R = \frac{V}{I}$ for any conductor
• Ohm’s Law says that for some conductors, current and voltage are proportional
  • Ohmic conductors (e.g. Resistors)
• For real conductors, that’s an approximation (e.g. $R = R(T)$ and $T = T(I)$)
Electric Power

• Fundamental application of Electricity
  - Deliver Electric Power
  - Converted to
    • Mechanical power
    • Heat
    • Light

Power = Energy/time =
dW/dt = (dq V)/dt =
dq/dt V = I V = I^2R = V^2/R
To keep charge moving
- work $W = q V$ to get from $d$ to $a$

Def: $\xi = \frac{\text{Work}}{\text{unit charge}}$
- $\xi$ is ‘Electromotive Force’ (EMF)

It’s not a Force!
- Units are $[V]$
- Sources of EMF: Battery, LVPS

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Electric Circuits

Resistor

\[ V_{ad} = V \]
\[ V_{ab} = 0 \]
\[ V_{cd} = 0 \]

Battery

\[ V_{bc} = V_{ad} = I R \]

Voltage Drop
Internal Resistance

\[ V_{ab} = \xi - I \cdot r \]

\[ V_{ab} = I \cdot R \]

\[ \Rightarrow \xi - I \cdot r = I \cdot R \]

\[ \Rightarrow I = \frac{\xi}{(r+R)} \]
DC Circuits

Resistors in series

\[ V_{ac} = V_{ab} + V_{ac} = I R_1 + I R_2 = I (R_1 + R_2) \]

\[ = I R_{eq} \quad \text{for} \quad R_{eq} = (R_1 + R_2) \]
DC Circuits

Resistors in parallel

\[ I = I_1 + I_2 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} = \frac{V_{ab}}{R_{eq}} \]

\[ \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]
Kirchoff’s Rules

• Junction rule

At junctions:

\[ \sum I_{in} = \sum I_{out} \]

Charge conservation

• Loop rule

Around closed loops:

\[ \sum \Delta V_j = 0 \]

\( \Delta V \) for both EMFs and Voltage drops

Energy conservation

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Kirchoff’s Rules

- Kirchoff’s rules allow us to calculate currents for complicated DC circuits
- Main difficulty: Signs!
- Rule for resistors:

\[ \Delta V = V_b - V_a = -I \cdot R \]

, if we go in the direction of \( I \) (voltage drop!)
Kirchoff’s Rules

• Kirchoff’s rules allow us to calculate currents for complicated DC circuits
• Main difficulty: Signs!
• Rule for EMFs:

\[ \Delta V = V_b - V_a = \xi, \text{ if we go in the direction of } I \]
Example

- Pick signs for $I_1$, $\xi$
- Junction rule
  \[ I_1 = 1A + 2A = 3A \]
- Loop rule (1)
  \[ 12V - 6V - 3A \cdot r = 0 \]
  \[ \Rightarrow r = \frac{6}{3} \Omega = 2 \Omega \]
- Loop rule (2)
  \[ 12V - 6V - 1V + \xi = 0 \]
  \[ \Rightarrow \xi = -5V \]
RC Circuits

- Currents change with time
- Example: Charging a capacitor

\[ t = 0 \]
\[ q = 0 \]
\[ V_c = \frac{q}{C} \]
RC Circuits

• Currents change with time
• Example: Charging a capacitor

![Diagram of an RC circuit]

\[ t = \text{infinity} \]
\[ q = C \, \xi \]
\[ V_c = \frac{q}{C} = \xi \]
RC Circuits

• What happens between $t=0$ and infinity?
RC Circuits

- What happens between t=0 and infinity?

\[ V_R = IR \]

\[ V_C = \frac{q}{C} \]
Charging capacitor

$$\xi - \frac{q}{C} - IR = 0 \quad \text{Loop Rule}$$

$$\xi - \frac{q}{C} - \frac{dq}{dt} R = 0 \quad \text{note } q(t=0) = 0, \quad I(t=0) = \frac{\xi}{R}$$

$$\Rightarrow \frac{dq}{dt} = -\frac{q}{RC} + \frac{\xi}{R} \quad \text{separate variables}$$

$$\Rightarrow \frac{dq}{q-\xi C} = -\frac{dt}{RC} \quad \text{integrate}$$

$$\Rightarrow \int_0^Q \frac{1}{q-\xi C} dq = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \ln\left(\frac{q-\xi C}{-\xi C}\right) = -\frac{t}{RC} \quad \text{exponentiate}$$

$$\Rightarrow \frac{q-\xi C}{-\xi C} = \exp\left(-\frac{t}{RC}\right)$$

$$\Rightarrow q(t) = \xi C\left[1 - \exp\left(-\frac{t}{RC}\right)\right]$$

$$\Rightarrow I(t) = -\frac{dq}{dt} = -\xi C \exp\left(-\frac{t}{RC}\right) \times \left(-\frac{1}{RC}\right)$$

$$\Rightarrow I(t) = \exp\left(-\frac{t}{RC}\right)$$

$$\Rightarrow V(t) = \frac{q(t)}{C} = \xi [1 - \exp\left(-\frac{t}{RC}\right)]$$

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Charging $C$

At $t = t_r$,  
$q(t) = (1 - \frac{1}{e}) q_{\text{max}} = 63\% q_{\text{max}}$

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In-Class Demo

Discharging a capacitor

\[ R \] \[ C \] \[ dq \] 

[Diagram of a circuit with a capacitor and resistor, showing charges +q and -q]
Discharging Capacitor

\[ \frac{-q}{C} - IR = 0 \quad \text{Loop Rule for} \quad \xi = 0, \quad q(t = 0) = Q_{\text{final}} = \xi C \]

\[ \frac{-q}{C} - \frac{dq}{dt}R = 0 \]

\[ \frac{dq}{dt} = \frac{R}{C} \]

\[ \Rightarrow \int_{Q_{\text{final}}}^{q} \frac{1}{q} dq = -\int_{0}^{t} \frac{dt}{RC} \]

\[ \Rightarrow \ln \left( \frac{q}{Q_{\text{final}}} \right) = -\frac{t}{RC} \]

\[ \Rightarrow q(t) = Q_{\text{final}} \exp \left( -\frac{t}{RC} \right) \]

\[ = \xi C \exp \left( -\frac{t}{RC} \right) \]

\[ \Rightarrow I(t) = \frac{dq}{dt} = \xi C \exp \left( -\frac{t}{RC} \right) \times \left( -\frac{1}{RC} \right) \]

\[ = -\frac{\xi}{R} \exp \left( -\frac{t}{RC} \right) \quad \text{note sign!} \]

\[ V(t) = \frac{q(t)}{C} = \xi \exp \left( -\frac{t}{RC} \right) \]

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Discharging C

\[ q(t) \]

\[ I(t) \]

\[ \tau = RC \]

\[ 37\% \]

\[ V(t) \]

\[ \tau = RC \]

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