Electricity and Magnetism

• Today
  - Charged particles in B-Field
  - Sources of B-Field
Magnetism

• Magnetic Force and Magnetic Field

• How are Force and Field related?

• What is the Source of the Field?
Force on moving charge

- Charged particle in a combination of Electric and Magnetic Field:

\[ \vec{F}_L = q \vec{E} + q \vec{v} \times \vec{B} \]

Lorentz Force

This we know!

Direction: Right-Hand Rule
**Force on moving charge**

In B-Field:

\[ F = m \ a \]

\[ |q| \ v \ B = m \ v^2/R \]

\[ \rightarrow \quad R = m \ v/(q \ B) \]

**Cyclotron Radius**

\[ \vec{F}_L = q \ E + q \ \vec{v} \times \vec{B} \]
Example

- Why so big?

\[ R = \frac{m v}{q B} \]

Momentum \( p \)

- Large \( p \) -> Large \( R \)
- \( B \sim 10T \) (very big!)
- Max. \( B \) in Lab around 100 T

Relativistic Heavy Ion Collider

Apr 3 2002
Work done on moving charge

\[ W = \mathbf{F}_E \cdot \mathbf{L} = q \mathbf{E} \cdot \mathbf{L} \]

\[ dW = \mathbf{F}_B \cdot d\mathbf{L} = (q \mathbf{v} \times \mathbf{B}) \cdot d\mathbf{L} = 0 \]
Magnet and Current

- Force on wire if $I \neq 0$
- Direction of Force depends on Sign of $I$
- Force perpendicular to $I$
Currents and Magnetic Field

Wire, I > 0
Currents and B-Field

- Current as Source of B
- Magnetic Field lines are always closed
  - no Magnetic Charge (Monopole)
- Right Hand Rule
Currents and B-Field
Currents and B-Field

- Superposition Principle!
Currents and B-Field

• Solenoid: Large, uniform B inside!
Current and Current

Attraction

Repulsion
Magnetic Field vs Electric Field

\[ \vec{B} = \frac{\mu_0}{(4 \pi)} \frac{Q}{r^2} \vec{v} \times \vec{r} \]

\[ \vec{E} = \frac{1}{(4 \pi \varepsilon_0)} \frac{Q}{r^2} \hat{r} \]

\[ \mu_0 = 4 \pi \cdot 10^{-7} \text{T m/A} \]

\[ \varepsilon_0 = 8.85 \cdot 10^{-12} \text{C}^2/(\text{Nm}^2) \]
Magnetic Field for Current $I$

\[
\mathbf{B} = \frac{\mu_0 I}{2\pi} \frac{\mathbf{dl} \times \mathbf{r}}{r^2}
\]
Magnetic Field for Current $I$

$$\vec{B} = \mu_0/(4 \pi) \frac{Q}{r^2} \vec{v} \times \hat{r} \quad \text{for single charge}$$

$I = \frac{dQ}{dt} \rightarrow dQ \vec{v} = dQ \frac{\vec{dl}}{dt} = I \vec{dl}$

$$d\vec{B} = \mu_0/(4 \pi) I \vec{dl} \times \hat{r}/r^2$$

Law of Biot-Savart:
Magnetic Field $d\vec{B}$ for current through segment $\vec{dl}$