Electricity and Magnetism

• Reminder
  – LC circuits / Oscillations
  – Displacement current
  – Maxwell’s equations

• Today
  – More on Maxwell’s equations
  – Electromagnetic waves

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LC Circuit

\[ \frac{d^2Q}{dt^2} + \frac{1}{(LC)} \cdot Q = 0 \]

\[ \omega_0^2 = \frac{1}{(LC)} \]

\[ \frac{d^2x}{dt^2} + \frac{k}{m} \cdot x = 0 \]

\[ \omega_0^2 = \frac{k}{m} \]

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LC Circuit

• Total energy $U(t)$ is conserved:

$Q(t) \sim \cos(\omega t)$

$dQ/dt \sim \sin(\omega t)$

$U_L \sim (dQ/dt)^2 \sim \sin^2$

$U_C \sim Q(t)^2 \sim \cos^2$

$\cos^2(\omega t) + \sin^2(\omega t) = 1$
Electromagnetic Oscillations

• In an LC circuit, we see oscillations:

\[
\begin{align*}
\text{Energy in E-Field} \\
\uparrow \\
\text{Energy in B-Field}
\end{align*}
\]

• Q: Can we get oscillations without circuit, i.e. when we have just the fields?

• A: Yes!
  - Electromagnetic Waves
Maxwell’s Equations (almost)

\[
\oint_{A_{\text{closed}}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]

\[
\xi = \oint_{L_{\text{closed}}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}
\]

\[
\oint_{A_{\text{closed}}} \vec{B} \cdot d\vec{A} = 0
\]

\[
\oint_{L_{\text{closed}}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}
\]

- Charges are the source of Electric Flux through close surface
- Changing magnetic field creates an electric field
- There are no magnetic monopoles
- Moving charges create magnetic field

- Connection between electric and magnetic phenomena
- But not symmetric
- -> James Clerk Maxwell (~1860)

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Displacement Current

• Ampere’s Law broken – How can we fix it?

\[ Q = C V \]

Displacement Current \( I_D = \varepsilon_0 \frac{d\Phi_E}{dt} \)

Changing field inside C also produces B-Field!
Displacement Current

- Example calculation: $B(r)$ for $r > R$

$$Q = C V$$

$$\Rightarrow B(r) = \frac{R^2}{2rc^2} \frac{dV}{dt}$$
Maxwell’s Equations

\[ \oint_{A_{\text{closed}}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \]

\[ \oint_{L_{\text{closed}}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]

\[ \oint_{A_{\text{closed}}} \vec{B} \cdot d\vec{A} = 0 \]

\[ \oint_{L_{\text{closed}}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \]

- M.E.’s **predict** electromagnetic waves, moving with speed of light
- Major triumph of science

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Electromagnetic Waves

• Until end of semester:
  – What are electromagnetic waves?
  – How does their existence follow from Maxwells equations?
  – What are the properties of E.M. waves?

• Prediction was far from obvious
  – No hint that E.M. waves exist
  – Involves quite a bit of math
Reminder on waves

• Examples of waves
  – Mechanical waves
  – Pressure waves
  – E.M. waves

• In-Class Demo...
Reminder on waves

At a moment in time:

\[ \text{Wavelength } \lambda \]

\[ \text{Amplitude} \]

\[ \text{Position } x \]

At a point in space:

\[ \text{Period } T = \frac{1}{f} \]

\[ \text{Amplitude} \]

\[ \text{Time } t \]
Reminder on waves

• Types of waves
  – Transverse
  – Longitudinal
    • compression/decompression
Reminder on waves

• For a travelling wave (sound, water)
  Q: What is actually moving?

• -> Energy!

• Speed of propagation: $v = \lambda f$

• Wave equation:

$$\frac{\partial^2 D(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D(x, t)}{\partial t^2}$$

Couples variation in time and space
Electromagnetic Waves

• Is light an electromagnetic wave?
  - Check speed and see if we can predict that
Back to Maxwell’s equation

- Wave equation is differential equation
- M.E. (so far) describe integrals of fields

\[
\begin{align*}
\oint_{A_{\text{closed}}} \vec{E} \cdot d\vec{A} &= \frac{Q_{\text{encl}}}{\varepsilon_0} \\
\oint_{L_{\text{closed}}} \vec{E} \cdot d\vec{l} &= -\frac{d\Phi_B}{dt} \\
\oint_{A_{\text{closed}}} \vec{B} \cdot d\vec{A} &= 0 \\
\oint_{L_{\text{closed}}} \vec{B} \cdot d\vec{l} &= \mu_0 I_{\text{encl}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}
\end{align*}
\]

Transform into differential eqn’s
Differential Form of M.E.

- Need two theorems: Gauss and Stokes
  - Gauss

\[ \mathbf{\nabla} \cdot \mathbf{F} dV = \oint_{A} \mathbf{F} \cdot d\mathbf{A} = \Phi_{F} \]

\[ \mathbf{\nabla} \cdot \mathbf{F} = \frac{\partial F_{x}}{\partial x} + \frac{\partial F_{y}}{\partial y} + \frac{\partial F_{z}}{\partial z} \]
Differential Form of M.E.

- Need two theorems: Gauss and Stokes
  - Stokes

\[ \oint_L \vec{F} \, d\vec{l} = \int_{A(L)} \nabla \times \vec{F} \, d\vec{A} \]

\[ \nabla \times \vec{F} = i \left( \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + j \left( \frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} \right) + k \left( \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \]
Differential Form of M.E.

\[ \oint_{A_{\text{closed}}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

\[ \oint_{L_{\text{closed}}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]

\[ \oint_{A_{\text{closed}}} \vec{B} \cdot d\vec{A} = 0 \]

\[ \oint_{L_{\text{closed}}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

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\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]

\[ \nabla \cdot \vec{B} = 0 \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \]
Differential Form of M.E.

\[
\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \\
\nabla \cdot \vec{B} = 0 \\
\n\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\
\n\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}
\]

In Vacuum
no charge

\[
\nabla \cdot \vec{E} = 0 \\
\n\nabla \cdot \vec{B} = 0 \\
\n\n\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\
\n\n\nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}
\]

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