Physics 8.03
Vibrations and Waves

Lecture 3
HARMONICALLY DRIVEN DAMPED
HARMONIC OSCILLATOR
Last time:

**Damped harmonic oscillator**

- **Equation of Motion**
  \[ \ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0 \]

- **Three solutions that depend on size of damping**
  
  \[
  x(t) = (A + Bt)e^{-\gamma t/2} \\
  = Ae^{-\gamma t/2} \cos(\omega t + \phi)
  \]

  - **Underdamped**
    \( \omega_0 > \frac{\gamma}{2} \)
  
  - **Critically damped**
    \( \omega_0 = \frac{\gamma}{2} \)

  - **Overdamped**
    \( \omega_0 < \frac{\gamma}{2} \)

- **Damping slows down natural frequency of oscillator**
  (or makes it stop oscillating altogether)

\[
\omega = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}
\]
DRIVEN HARMONIC MOTION

- Add driving force term to equation of motion
- Effect
  - Oscillator loses its own identity and oscillates at the frequency at which it is driven (not its own natural frequency)
- Mathematical solution
  - Amplitude of oscillation depends on driving frequency
  - Phase of oscillation (relative to driving force) also depends on driving frequency
  - When driving frequency = natural frequency
    - RESONANCE!
- Examples ➔ shattering a wine glass with sound
Next time: Transient behavior

- What happens when driving force is first turned on? Transients
- We started with a second order diff. eqn. so we should get two constants of integration. Where are they?
- Complete solution to the diff. eqn. includes the homogeneous solution (we got that today) AND a particular solution (that describes the transient behavior of the driven oscillator)