Last time:

External driving force

- Applied an external driving force to a coupled oscillator system
  - In steady-state coupled system takes on frequency of the driving force
  - When driving force is at a normal mode frequency ➔ resonance
A Recipe’ for coupled oscillators

- Find forces acting on each particle
- Coupled differential equations
  - No driving force $\rightarrow$ homogeneous
  - Driving force $\rightarrow$ at least one eqn. is inhomogenous
- Always solve homogeneous equation first
- Trial solution $\rightarrow x_i(t) = C_i \cos(\omega t - \delta)$
- Coupled (simultaneous) algebraic equations

$C = D$
A Recipe’ for Coupled Oscillators

...contd...

- “Normal” modes
  - Frequencies (eigenvalues): \( \omega_i \) are the roots of \( \lambda^2 \), calculate by solving for \( \omega \) when \( \det(\lambda^2) = 0 \)
  - Ratios of amplitudes: Plug \( \omega = \omega_i \) back into \( \lambda \rightarrow C \)

- Any other motion \( \Rightarrow \) superposition of all normal modes

- Now turn on the harmonic driving force

- Solve inhomogenous set using Cramer’s rule
  - For each \( C_i \), replace the \( i \)-th column of \( \lambda \) with \( \vec{D} \)
Last time: $N$ coupled oscillators

- $N$ identical oscillators ($N$ beads on a string)
  - $N$ normal modes
  - Frequency and amplitude of motion of the $p$-th depends on
    - Mode number, $n$
    - Location of particle in the array, $p$

- As $N \rightarrow \infty$, we get a continuous system of oscillators
Wave Equation and its Solutions

- **Waves** ➔ oscillations in space and time
  - $y(x, t)$
  - Transverse or longitudinal waves
  - Traveling or standing waves

- **Solutions to wave equation**
  - Pulses of arbitrary shape ➔ $y(x, t) = f(x \pm vt)$
  - Harmonic pulses ➔ $y(x, t) = y_0 \cos(k(x \pm vt) + \phi)$
  - Separable solutions ➔ $y(x, t) = f(x) \cos(\omega t + \phi)$