Quiz 1

Name: (Last, First) ____________________________ (please print).

Recitation number (circle one):  1  2  3

• Record all answers and show all work in this exam booklet. If you need extra space, use the back of the opposing page.
• All scratch paper must be handed in with the exam, but will not be graded.
• No materials besides pencils and erasers are allowed (no calculators, notes, books, pets, etc.)
• Whenever possible, try to solve problems using general analytic expressions. Plug in numbers only as a last step.
• Please make sure to answer all sub-questions.
• Good luck!

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<th>Problem</th>
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(a). **(4 pts)** Professor F. Ishy claims to have discovered a new force of magnitude $F = A|\mathbf{r}_1 + \mathbf{r}_2|$ between protons, where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of the protons and $A$ is a constant that he's trying to get named in his honor.

(A) Is this force translationally invariant? YES / NO (circle one)

(B) Is this force invariant under rotations around $\mathbf{r} = 0$? YES / NO (circle one)

(b). **(4 pts)** Professor C. Rank claims that a charge at $(\mathbf{r}_1, t_1)$ will contribute to the air pressure at $(\mathbf{r}_2, t_2)$ by an amount $B \sin[C(||\mathbf{r}_2 - \mathbf{r}_1||^2 - c^2|t_2 - t_1|^2)]$, where $B$ and $C$ are constants.

(A) Is this effect Galilean invariant? YES / NO (circle one)

(B) Is this effect Lorentz invariant? YES / NO (circle one)

(c). **(1 pt)** To 1 significant figure, the speed of light in meters/second is ____________

(d). **(2 pts)** From which two postulates did Einstein derive special relativity?

(e). **(11 pts)** Indicate whether each of the following statements are true of false.

(A) The proper length of a ruler is Lorentz invariant. TRUE / FALSE (circle one)

(B) The wave equation is Galilean invariant. TRUE / FALSE (circle one)

(C) The kinetic energy of a particle is Lorentz invariant. TRUE / FALSE (circle one)

(D) The acceleration of a particle is Galilean invariant. TRUE / FALSE (circle one)

(E) A ping-pong ball moving near the speed of light still looks spherical. TRUE / FALSE (circle one)

(F) X-rays travel faster than microwaves. TRUE / FALSE (circle one)

(G) If you could send a signal faster than light, then there’s a frame where you could send a signal backward in time. TRUE / FALSE (circle one)

(H) If two twins are reunited, who is oldest may be frame-dependent. TRUE / FALSE (circle one)

(I) No experiment inside an isolated sealed lab in space can determine its orientation. TRUE / FALSE (circle one)

(J) No experiment inside an isolated sealed lab in space can determine its velocity. TRUE / FALSE (circle one)

(K) No experiment inside an isolated sealed lab in space can determine its acceleration. TRUE / FALSE (circle one)

(f). **(3 pts)** List three pieces of observational evidence supporting special relativity.
Question 2: Pole Vaulter Deluxe

Your goal in this problem is to completely fill out the table below.

<table>
<thead>
<tr>
<th></th>
<th>S (Chris)</th>
<th>S′ (Zoe)</th>
<th>S″ (Train)</th>
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<tr>
<td>x_B</td>
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<td>ct_C</td>
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To save time, note that all three frames have the same spacetime origin (at event A) and that there is no need to draw spacetime diagrams.

(a). (2 pts) Based on the text below, fill in the entries corresponding to X_B and X_C′ in the table above (no calculation needed!). Chris is standing next to a barn taking measurements as Zoe runs through the barn in the x-direction holding a pole horizontally in the direction of motion. In Zoe’s frame S′, the pole has length L, the event when the rear of the pole is aligned with the entrance to the barn is X_A′ = (x_A′, ct_A′) = (0, 0), and the location of the front of the pole at that same time is X_C′ = (L, 0). In Chris’ frame S, Zoe is running at speed β_1, the rear of the pole is aligned with the entrance to the barn (event A) at X_A = (x_A, ct_A) = (0, 0), and the front of the pole is aligned with the barn exit (event B) at X_B = (x_B, ct_B) = (L/γ_1, 0), where γ_1 = 1/√(1−β_1^2).

(b). (7 pts) Compute X_B and X_C and fill in the corresponding entries in the table above.

(c). (8 pts) A train with the rest of the 8.033 students passes by, and Chris measures its speed to be β_2 in the x-direction. The train’s frame S″ is aligned such that X_A″ = (0, 0). Compute X_B″ and X_C″ in terms of L, γ_1, β_1, γ_2, and β_2, and fill in the corresponding entries in the table above.

(d). (4 pts) Show that for β_2 = 0, the students on the train agree with Chris.

(e). (4 pts) Show that for β_2 = β_1 the students on the train agree with Zoe.
Question 3: Variational calculus: Newton’s second law modified

For a particle of rest mass \( m_0 \) in a potential \( V(x) \), show that out of all trajectories \( x(t) \) between two events \( A \) and \( B \), the one maximizing the quantity

\[
\int_{t_A}^{t_B} \left[ \frac{1}{\gamma} + \frac{V(x)}{m_0c^2} \right] dt
\]

satisfies the relation

\[
\frac{d}{dt} (m_0\gamma\dot{x}) = -V'(x)
\]

(the relativistic version of Newton’s second law). Here \( \gamma \equiv 1/\sqrt{1 - \dot{x}^2/c^2} \), \( \dot{x} \equiv dx/dt \) and \( V' = dV/dx \).
Question 4: Relativistic Red Sox

In a parallel universe, the Boston team made the playoffs.

(a) (6 pts) Manny Relativirez hits the ball and starts running towards first base at speed $\beta$. How fast is he running, given that he sees third base 45° to his left (as opposed to straight to his left before he started running)? Assume that he is still very close to home plate.

(b) (7 pts) A player standing on third base is wearing red socks emitting light of wavelength $\lambda_{\text{red}}$. What wavelength does Manny see? What color are the socks according to Manny, in the approximation that $\lambda_{\text{green}} = \lambda_{\text{red}}/2^{1/4}$, $\lambda_{\text{blue}} = \lambda_{\text{red}}/\sqrt{2}$, $\lambda_{\text{violet}} = \lambda_{\text{red}}/\sqrt{3}$?

(c) (5 pts) In Manny’s frame, the ball is moving with speed $c/\sqrt{2}$ towards first base. How fast is it going in the rest frame of the stadium?

(d) (5 pts) Later in the game, a squabble erupts. An outfielder catches a fly ball at $(x_A, y_A, z_A, ct_A) = (50\text{m}, 40\text{m}, 2\text{m}, cT)$ (event A) and Manny starts to run from second base at $(x_B, y_B, z_B, ct_B) = (30\text{m}, 30\text{m}, 0, 0)$ (event B), where $cT = 30\text{m}$, which corresponds to $T \approx 90$ nanoseconds. The outfielder claims that Manny is out by virtue of running too soon, whereas Manny claims that B preceded A is his frame. You are are the umpire and must settle this. Compute the spacetime interval between A and B and indicate whether it is (circle one) TIMELIKE / SPACELIKE / NULL

(e) (2 pts) Is Manny correct? YES / NO / DEPENDS ON HIS VELOCITY (circle one) In one sentence, why?