Welcome back to 8.033!

Emmy Noether
1882-1935

Image courtesy of Wikipedia.
PRACTICAL STUFF:

• PS1 due Friday 4PM
• Symmetry notes posted

TODAY’S TOPIC: SYMMETRY IN PHYSICS

• Key concepts: frame, inertial frame, transformation, invariant, invariance, symmetry, relativity

• Key people: Galileo Galileo, Emmy Noether

• Symmetry examples: translation, rotation, parity, boost

• Million Dollar question: what are the symmetries of physics?
What do we mean by symmetry?
WHAT'S THE SYMMETRY OF THE UNIVERSE?

OF PHYSICS?
Invariance under translation

- No experiment within your lab can determine whether it’s been shifted
- Original frame: masses at $r_1$ and $r_2$.
  \[ F = \frac{GmM}{|r_2 - r_1|^2} \]
- Primed frame: masses at $r'_1 = r_1 + a$ and $r'_2 = r_2 + a$.
  \[ F' = \frac{GmM}{|r'_2 - r'_1|^2} = \frac{GmM}{|(r_2 + a) - (r_1 + a)|^2} = \frac{GmM}{|r_2 - r_1|^2} = F \]
Invariance under rotation

- No experiment within your spaceship can determine whether it’s been rotated.
- Is everyone cool with $3 \times 3$ matrices?
- Primed frame: masses at $r'_1 = Rr_1$ and $r'_2 = Rr_2$

$$F' = \frac{GmM}{|Rr_2 - Rr_1|^2} = \frac{GmM}{|R(r_2 - r_1)|^2} = \frac{GmM}{r_2 - r_1|^2} = F$$

- Another example: Maxwell’s equations in vacuum imply

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \ddot{\mathbf{E}}.$$ 

Since only differences in position and time enter, it’s translationally invariant. Here it’s infinitesimal differences (derivatives), above it was a finite difference $|r_2 - r_1|^2$.

- $\nabla^2$ is invariant under rotation (remember Gauss’ theorem)

- At MIT:

$$\mathbf{E} = \frac{1}{c^2} \ddot{\mathbf{E}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$ 

- Near Australia:

$$\mathbf{E} = \frac{1}{c^2} \ddot{\mathbf{E}} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}.$$ 

- So both observer’s agree that Maxwell was right, i.e., the wave equation is translationally and rotationally invariant.
Invariance under reflection?

- Yes for all of classical physics
- Considered self-evident and obvious
- 1956: Chen Ning Yang & Tsung-Dao Lee propose that weak interactions violate parity; Chien-Shiung Wu demonstrates it with cobalt 60, Leon Lederman with accelerator. (Yang & Lee get 1957 Nobel prize.)
Symmetry is at the heart of modern physics

• Special relativity is all about so-called Lorentz symmetry.

• General relativity is about so-called diffeomorphism symmetry.

• Key topics in particle physics are C, P and T symmetry and combinations like CP and CPT symmetry.

• A cornerstone of particle physics is gauge symmetry

• In 2007, the Large Hadron Collider at CERN will search for super-symmetry.
WHAT’S THE SYMMETRY OF CLASSICAL MECHANICS?
Invariance under Galilean transformation

- Demo with colliding carts, ball.
- So Newtonian mechanics appears to be invariant - let’s understand exactly what the transformation is, and why this is so.
- Inertial frame definition \((a = 0 \text{ if } F = 0)\)
- Are we in an inertial frame? (PS1)
- Galilean transformation definition (between 2 inertial frames)
- Definition of event: a 4D point \((x, y, z, t)\). Examples?
- \(r' = r - vt\)
- Lengths invariant: \(\Delta r' = r'_2 - r'_1 = (r_2 - vt) - (r_1 - vt) = \Delta r\)
- But we must measure \(r'_1\) and \(r'_2\) at the same time!
- Which we can, since time is invariant and unambiguous: \(t' = t\)
Spacetime transformation summary

- Translation:
  \[
  \begin{aligned}
  r' &= r + \Delta r \\
  t' &= t + \Delta t
  \end{aligned}
  \]

- Rotation:
  \[
  \begin{aligned}
  r' &= Rr \\
  t' &= t
  \end{aligned}
  \]

- Galilean:
  \[
  \begin{aligned}
  r' &= r + vt \\
  t' &= t
  \end{aligned}
  \]

- Combined:
  \[
  \begin{aligned}
  r' &= Rr + \Delta r + vt \\
  t' &= t + \Delta t
  \end{aligned}
  \]
Transforming velocity

• How does \( u \) transform under a Galilean transformation?

\[
\begin{align*}
    u & \equiv \frac{dr}{dt} \\
    u' & \equiv \frac{dr'}{dt'} = \frac{d}{dt}(r - vt) = \frac{dr}{dt} - v = u - v
\end{align*}
\]

So velocities add/subtract as you’d expect: \( u' = u - v \)

• But what about the flashlight on the train?

Transforming acceleration

\[
\begin{align*}
    a & \equiv \frac{du}{dt} \\
    a' & \equiv \frac{du'}{dt'} = \frac{d}{dt}(u + v) = \frac{du}{dt} = a
\end{align*}
\]

So acceleration is invariant.
Transforming $F = ma$

- Consider forces that depend on separation:
  - Spring: $F = k(x_2 - x_1)$
  - Gravity: $F = \frac{GmM}{|r_2 - r_1|^2}$

  They are invariant, since lengths are.

- $m$ is invariant

- Since $F$, $a$ and $m$ are all invariant, so is the equation $F = ma$.

- So the physical law is invariant, but not the initial conditions!
Transforming energy & momentum

- Neither is invariant, since \( v \) isn't.

- But the conservation laws are invariant: \( E \) and \( p \) are conserved in any frame (PS1).

- Work-energy theorem:

  \[
  W = \Delta KE,
  \]

  where work defined as

  \[
  W = \int_{x_1}^{x_2} F dx.
  \]

- Proof:

  \[
  W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} ma \, dx = m \int_{x_1}^{x_2} \frac{dv}{dt} \, dx
  \]

  \[
  = m \int_{v_1}^{v_2} \frac{dx}{dt} \, dv = m \int_{v_1}^{v_2} v \, dv = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \Delta KE.
  \]

  Only assumption here was \( F = ma \), which is invariant, so the work-energy theorem is also invariant.

- \( W \) and \( KE \) alone are not invariant.
Transforming trajectories

- Is the 3D shape of a trajectory not invariant?
- No! Basket ball example: line in frame A is parabola in frame B.

Key Galilean non-invariants

\[
\begin{align*}
    r' &= r + vt \\
    u' &= u + v \\
    p' &= p + mv
\end{align*}
\]
SO WHICH DO YOU TRUST MORE:
Classical Mechanics, or E&M?