Problems

Problem 10.1 (20 pts)

A point source of light, $S$, of frequency $\nu$ is placed in the air ($n = 1$) a distance $L$ from a perfect conductor as shown in Figure 1.

![Figure 1: Point light source](image)

a. For what values of $L$ will the observer see a maximum in the interference pattern a long distance from $S$ in a direction perpendicular to the perfect conductor? (You may ignore any absorption by $S$ of the reflected light).

b. For what values of $L$ will the observer see a minimum in the interference pattern a long distance from $S$ in a direction perpendicular to the perfect conductor? (You may ignore any absorption by $S$ of the reflected light).

c. What is the minimum thickness of glass, of refractive index $n$, which must be placed between the light source and the mirror, so that for a given $L$ a minimum replaces a maximum, and vice versa? (You may assume there is no reflection from the glass surface)
Problem 10.2 (20 pts)

Three long narrow slits shown in Figure 2 are illuminated by a monochromatic source of light with wavelength $\lambda = 500$ nanometers ($1 \text{ nm} = 10^{-9} \text{ m}$). The distance between slit centers is $d = 1$ millimeter. The transmitted light is projected on a screen $L = 5$ meters away. Assume that the slits are very narrow such that the single slit diffraction can be ignored.

![Figure 2: Three slit screen (vertical and horizontal dimensions are not to scale)](image)

Consider first a case when the top slit is completely blocked.

a. What is the distance in millimeters between the central intensity maximum (fringe) and the first minimum on the screen?

b. Sketch the intensity distribution near the central fringe including total of 5 neighboring intensity maxima (central and two on each side). Label carefully the axes. What is the maximum intensity of each of the 5 fringes if single slit intensity is $I_0$?

The top slit is now uncovered.

c. What is now the distance in millimeters between the central maximum (fringe) and the first minimum on the screen?

d. Sketch the new intensity distribution near the central fringe including total of 5 neighboring maxima (central and two on each side) as a function of distance from the center of central peak $x$. Label carefully the axes.

e. What is the maximum intensity of each of the 5 fringes?

The top slit is now covered with a magic material that transmits only half of the incident light intensity without affecting phase, polarization or any other properties of light.

f. What is the maximum intensity that a fringe can have in this configuration? Which of the 5 neighboring fringes will have this intensity?

g. Write an equation in terms of fringe position $x$ that can be solved to find position of the intensity minimum nearest to the central peak. Do not solve.

h. Sketch now the new intensity distribution near the central fringe including total of 5 neighboring maxima (central and two on each side).
Problem 10.3 (20 pts)

Two dipole radiators are separated by a distance $\lambda/2$ along the $x$-axis. The dipoles are oriented along $z$. Assume $r >> \lambda$. See Figure 3.

![Figure 3: Two dipoles](image)

a. Find the relative intensities of the radiation in the $x - y$ plane at $\theta = 0$, $\pi/3$, $\pi/2$, and $\pi$ if the oscillations of the dipoles are in phase.

b. Repeat a. if the oscillators are out of phase by $\pi$.

c. The oscillators are spaced by a distance $\lambda/4$ and they are $\pi/2$ out of phase. Find the relative intensities at $\theta = 0$, $\pi/2$, $3\pi/2$

Problem 10.4 (20 pts)

A small amount of oil ($n = 1.45$) drops on the smooth surface of a lake ($n = 1.33$). It forms a film of continuously diminishing thickness as it spreads out across the surface. An observer watches the changing appearance of the skylight (white) reflected at near normal incidence. See the wavelengths in Table 1.

a. Initially, when the film is thick, the reflected light appears white to the naked eye. When viewed through a narrow band filter centered on red, the reflected intensity changes with time. Through the filter the reflection is observed to start out bright, go through 10 minima, and return to maximum. How much has the thickness of the film changed during this time?

b. Only when the film has thinned sufficiently does it appear colored to the unaided eye. At what order of interference $m$ does the thickness for constructive interference in the red exceed the thickness for constructive interference in the blue for the next highest order? (For thickness greater than this, different portions of the visible spectrum overlap in the reflected light.)

c. Just when the film on the flat surface appears green, a wind generated surface wave crosses the field of view. As a result the color oscillates with time. Toward which end of spectrum does the color change? (Recall that for thin film interference, the optical path length difference is proportional to the cosine of the angle the ray makes with normal in the film.)
### Table 1: Wavelengths of visible light.

<table>
<thead>
<tr>
<th>Color</th>
<th>Wavelength [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>white</td>
<td>all visible wavelengths</td>
</tr>
<tr>
<td>blue</td>
<td>480</td>
</tr>
<tr>
<td>green</td>
<td>530</td>
</tr>
<tr>
<td>red</td>
<td>650</td>
</tr>
<tr>
<td>black</td>
<td>no visible wavelength</td>
</tr>
</tbody>
</table>

**Problem 5 (20 pts)**

Consider a screen with two long parallel slits. Slit width is \( a \) and the distance between the two slits is \( b \). They are illuminated by a parallel beam of light with wavelength \( \lambda \) as shown in Figure 4.

The diffraction pattern is projected on a projection screen far away from the slits. The distribution of light on the screen as a function of \( \sin \psi \) is shown in Figure 5.

a. Estimate the values of \( a \) and \( b \) in units of \( \lambda \).

b. Now the slits are widened such that the distance between the slits is doubled to \( 2b \) while keeping the slit width \( a \) the same. Sketch the light intensity as a function of \( \sin \psi \). Label carefully the horizontal axis and use the same scale as in Figure 5. Vertical axis is arbitrary, we are asking only about the shape of the pattern and not its absolute magnitude.
Figure 5: Interference and diffraction pattern