8.03 Lecture 21

Last lecture:
*Thin film interference: We explained why soap bubbles are colorful.
*We will learn about:

1. Interference phenomenon with double-slit experiment: laser, water ripple
2. How phased radar works (radio waves 3 kHz – 300 GHz)
3. Connection to quantum mechanics

*Reminder: Huygens Principle:
All points on a wave front become a source of a spherical waves.

That works for odd spatial dimension and can be derived from Maxwell’s equations.
Last time: Double-Slit experiment:
where $L \gg d$

Optical path length difference:

$$r_B - r_A = d \sin \theta$$

Phase difference:

$$\delta = \frac{d \sin \theta}{\lambda} 2\pi = (d \sin \theta)k$$

What is the resulting intensity?

First: write down the electric field in complex notation.

$$\vec{E} = \vec{E}_A + \vec{E}_B = \left( E_0 e^{i(\omega t - kr_A)} + E_0 e^{i(\omega t - kr_B)} \right) \hat{z}$$

$$= E_0 e^{i(\omega t - kr_A)} \left[ 1 + e^{-i\delta} \right] \hat{z}$$

$$= E_0 e^{i(\omega t - kr_A)} e^{-i\delta/2} \left[ e^{i\delta/2} + e^{-i\delta/2} \right] \hat{z}$$

$$= 2 \cos(\delta/2) \hat{z}$$

$$\langle I \rangle \propto |\vec{E}|^2 = E \cdot E^* \propto \cos^2 \left( \frac{\delta}{2} \right)$$

$$\langle I \rangle = A \cos^2 \left( \frac{\delta}{2} \right)$$

Where $A$ is the intensity at $\delta = 0$

$$\sin \theta = \frac{\lambda}{2\pi d} \delta$$

Now we have the knowledge we need to understand how radars work!! Consider a triple-slit interference experiment:
\[ \delta_{12} = \delta_{23} = d \sin \theta = \delta \]

What is the required minimum \( \delta \) to have destructive interference?

\[ \Rightarrow \delta = \frac{2\pi}{3} \]

How about 4-slit?

\[ \Rightarrow \delta = \frac{2\pi}{4} = \frac{\pi}{2} \]

For a 5-slit experiment the minimum delta would be \( \frac{2\pi}{5} \) and so on. You can see that the width of the intensity peak is DECREASING as we increase the number of slits!

N-slit (N source) interference:
\[ \delta = d \sin \theta \cdot k \]

\[
E_{\text{Total}} = E_0 \left[ e^{i(\omega t - kR)} + e^{i(\omega t - kR - \delta)} + e^{i(\omega t - kR - 2\delta)} + \ldots + e^{i(\omega t - kR - (N-1)\delta)} \right]
\]

\[
= E_0 e^{i(\omega t - kR)} \frac{1 + e^{-i\delta} + e^{-2i\delta} + \ldots + e^{-(N-1)i\delta}}{1 - e^{-i\delta}} \\
= \sum_{m=0}^{N-1} (e^{-i\delta})^m
\]

\[
= E_0 e^{i(\omega t - kR)} \left( \frac{1 - e^{-i\delta N}}{1 - e^{-i\delta}} \right)
\]

\[
= E_0 e^{i(\omega t - kR)} \left( \frac{e^{-i\delta N/2}(e^{+i\delta N/2} - e^{-i\delta N/2})}{e^{-i\delta/2}(e^{+i\delta/2} - e^{-i\delta/2})} \right)
\]

\[
= E_0 e^{i(\omega t - kR)} e^{-i(\delta(N-1)/2)} \frac{\sin(N\delta/2)}{\sin(\delta/2)}
\]

\[
\langle I \rangle \propto |\vec{E}|^2 = \vec{E} \cdot \vec{E}^* \Rightarrow \langle I \rangle = I_0 \left[ \frac{\sin(N\delta/2)}{\sin(\delta/2)} \right]^2
\]
1. At $\delta = 0$

2. As we increase $\delta$

3. $\delta = \frac{2\pi}{N}$

$N$-radiators $\Rightarrow N - 2$ secondary maximum. Width of principle maximum: $\frac{4\pi}{N} \propto \frac{1}{N}$

Corresponding resolution:
\[
\frac{d \sin \theta}{\lambda} = \frac{2\pi}{N} \quad \sin \theta = \frac{2\pi \lambda}{Nd}
\]

We learn that: to get high resolution (i.e. small \( \theta \))

1. Use small \( \lambda \)
2. Large \( d \)
3. Large \( N \)

Sweep?
If we want a sweep frequency \( \phi \) we add additional phase difference between the sources \( \Delta \phi = \phi \cdot t \)

\[
\delta = \frac{2\pi}{\lambda} d \sin \theta - \phi \cdot t
\]

Where the first term is the phase difference from the optical path length and the second term is the additional phase difference from the source. The Principle Maximum happens at \( \delta = 0 \) or

\[
\sin \theta = \frac{\phi t \lambda}{2\pi}
\]

N source Phased Radar:
Where we have the additional phase differences on the left which change the direction of the principle maximum. We see interference: light, water, sound, ...
Single Electron Experiment:

 Emit one electron every time.

(1) No interference

An electron interferes with itself
(Predicted by Quantum Physics!)

(2) Interference
We learned the interference of two EM waves to N EM waves.

We call the interference of infinite number of EM waves “diffraction”.

We have $\infty$ point like spherical EM wave sources. This situation: we will see the “interference” between all the spherical wave sources.

Feynman: No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage.

Usually we use “interference” when we are talking about a few sources and “diffraction” when we are talking about many sources.
We can also divide the slit into 3 pieces.

\[
\delta = \frac{D}{2} \sin \theta \frac{2\pi}{\lambda}
\]

Destructive interference: \(\delta = \pi\)

\[\Rightarrow \sin \theta = \frac{\lambda}{D} \cdots \text{minimum!}\]

Destructive
\[
\delta = \frac{D}{3} \sin \theta \frac{2\pi}{\lambda} = \frac{2\pi}{3}, \frac{4\pi}{3}
\]

\[\Rightarrow \sin \theta = \frac{\lambda}{D}, \frac{2\lambda}{D}, \cdots \]

Divide into \(N\) pieces

\[\Rightarrow \sin \theta = \frac{\lambda}{D}, \frac{2\lambda}{D}, \cdots, \frac{(N-1)\lambda}{D}\]