Review:

Last lecture:

* Thin film interference

  Explained why soap bubbles are colorful

* We will learn about:

  1. Interference phenomenon with double-slit experiment: laser, water ripple
  2. How phased radar works. (radio waves, time delay)
  3. Connection to Quantum Mechanics.

* Reminder: Huygens Principle:

  All point on a wave front becomes a source of a spherical wave.

  \[ \text{Works for odd spatial dimension, can be derived from Maxwell's Eqs.} \]
Last time:

**Double-Slit Experiment:**

![Diagram of double-slit experiment with equations]

**Optical Pathlength Difference:**

\[ \gamma_B - \gamma_A = d \sin \theta \]

\[ \Rightarrow \text{Phase difference:} \]

\[ \delta = \frac{d \sin \theta}{\lambda} \cdot 2\pi \]

\[ = (d \sin \theta) \cdot k \]

What is the resulting intensity?

First: write down the electric field (in the complex notation)

\[ \mathbf{E} = \mathbf{E}_A + \mathbf{E}_B = (E_0 e^{i(wt-k\gamma_A)} + E_0 e^{i(wt-k\gamma_B)}) \]

\[ = E_0 e^{i(wt-k\gamma_A)} \left[ 1 + e^{-i\delta} \right] \]

\[ = E_0 e^{i(wt-k\gamma_A)} e^{-i\frac{\delta}{2}} \left[ e^{i\frac{\delta}{2}} + e^{-i\frac{\delta}{2}} \right] \]

\[ = E_0 e^{i(wt-k\gamma_A)} e^{-i\frac{\delta}{2}} \left[ 2 \cos \frac{\delta}{2} \right] \]

\[ \langle I \rangle \propto |E|^2 = E \cdot E^* \propto \cos^2 \frac{\delta}{2} \]

\[ \Rightarrow \langle I \rangle = A \cos^2 \frac{\delta}{2} \]

\[ \uparrow \text{the intensity at } \delta = 0 \]
\[
\sin \theta = \frac{\lambda}{2\pi d} \delta 
\]

\( \delta = 0 \) \quad \rightarrow \quad \vec{E}_B \rightarrow \vec{E}_A \quad \Rightarrow \quad \delta = 2\pi \quad \rightarrow \quad \vec{E}_B \rightarrow \vec{E}_A 

Now we have the knowledge we need to understand how radars work!!

Consider a three-slit interference exp:

\( S_{12} = S_{23} = d \sin \theta = \delta \)

(minimum)

What is the required \( \delta \) to have destructive interference?

\( \Rightarrow \quad \delta = \frac{2\pi}{3} \)

How about 4-slit?

\( S = \frac{2\pi}{4} = \frac{\pi}{2} \)

5-slit?

\( S = \frac{2\pi}{5} \)

\( \Rightarrow \) You can see that the width of the intensity peak is DECREASING as we increase the number of slits!!
For the N-slit interferometer, the total electric field is given by:

\[ E_{\text{total}} = E_0 \left[ e^{i(wx-kr)} + e^{i(wx-kr-k)} + \cdots + e^{i(wx-kr-(N-1)s)} \right] \]

\[ = E_0 e^{i(wx-kr)} \left[ 1 + e^{-is} + e^{-2is} + \cdots + e^{-i(N-1)s} \right] \]

\[ = E_0 e^{i(wx-kr)} \left( \frac{1 - e^{-isN}}{1 - e^{-is}} \right) \]

\[ = E_0 e^{i(wx-kr)} \frac{e^{-i\frac{\pi N}{2}}}{e^{-i\frac{\pi}{2}} (e^{i\frac{\pi}{2}} - e^{-i\frac{\pi}{2}})} \]

\[ = E_0 e^{i(wx-kr)} \frac{e^{-i\frac{\pi N}{2}}}{e^{i\frac{\pi}{2}} (e^{i\frac{\pi}{2}} - e^{-i\frac{\pi}{2}})} \]
Therefore \[ E_{\text{Total}} = E_0 e^{i(\omega t - KR)} e^{-i \left( \frac{S(N+1)}{2} \right)} \frac{\sin \left( \frac{NS}{2} \right)}{\sin \left( \frac{S}{2} \right)} \]

\[ \langle I \rangle \propto |E|^2 = \mathbf{E} \cdot \mathbf{E}^* \]

\[ \Rightarrow \langle I \rangle = I_0 \left[ \frac{\sin \left( \frac{NS}{2} \right)}{\sin \left( \frac{S}{2} \right)} \right]^2 \]

Ex: \( N = 7 \)

Principle Maximum

Secondary Maximum

At \( S = 0 \)

2. Increase \( S \)

3. \( S = \frac{2\pi}{N} \)

N-radiators \( \Rightarrow \) N-2 secondary maximum

Width of principle maximum: \( \frac{4\pi}{N} \times \frac{1}{N} \)
Corresponding resolution:

\[
\frac{d\sin \theta}{\lambda} = \frac{2\pi}{N}
\]

\[
\sin \theta = \frac{2\pi \lambda}{Nd}
\]

We learn that to get high resolution (small \(\theta\))

1. Use small \(\lambda\)
2. Large \(d\)
3. Large \(N\)

Sweep \(\phi\)?

If we want a sweep frequency \(\phi\)

\[\Rightarrow \text{Add additional phase difference between the sources} \Delta \phi = \phi \cdot t\]

\(t = 0\)

\(t = 1\)

\(\phi_0 + \phi\): original phase
\[ S = \frac{2\pi}{\lambda} \cdot d \sin \Theta - \phi \cdot t \]

Phase difference Additional Phase difference from optical path length difference.

\[ \Rightarrow \text{Principle maximum: } S = 0 \]

\[ \Rightarrow \sin \Theta = \frac{\phi + \lambda}{2\pi} \]

N source Phased Radar:

6\phi  
5\phi  
4\phi  
3\phi  
2\phi  
\phi   
0

Additional phase difference \( \Rightarrow \) change the direction of the principle maximum
We see interference: light, water, sound, ...

Single Electron Experiment:

Emit one electron every time.

(1) No interference

(2) Interference

An electron interferes with itself
(Predicted by Quantum Physics!)
We learned the interference of 2 EM waves by N EM waves.

Interference of infinite number of EM waves.

"Diffraction"

We have ∞ point like spherical EM wave sources.

This situation: we will see the "interference" between all the spherical wave sources.

We call it "diffraction".

Feynman: No one has ever been able to define the difference between interference and diffraction satisfactorily.

It is just a question of usage.
Usually: Interference: a few sources
Diffraction: many sources

\[ S = \frac{D}{2} \sin \theta \frac{2\pi}{\lambda} \]
Destructive Int: \[ S = \pi \]
\[ \Rightarrow \sin \theta = \frac{\lambda}{D}, \quad \text{minimum!} \]

We can also divide the slit into 3 pieces

\[ S = \frac{D}{3} \sin \theta \frac{2\pi}{\lambda} = \frac{2\pi}{3}, \frac{4\pi}{3} \]
\[ \Rightarrow \sin \theta = \frac{\lambda}{D}, \frac{2\lambda}{D} \]

divide into \( N \) pieces
\[ \Rightarrow \sin \theta = \frac{\lambda}{D}, \frac{2\lambda}{D}, \frac{3\lambda}{D}, \ldots \frac{(N-1)\lambda}{D} \]

Next time: Intensity.