8.03 Lecture 22

We learned the interference of two EM waves to N EM waves.

![Diagram of EM wave interference](image1)

We call the interference of infinite number of EM waves “diffraction”.

![Diagram of diffraction](image2)

We have ∞ point like spherical EM wave sources. This situation: we will see the “interference” between all the spherical wave sources. We call it “diffraction”.

Feynman: No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage.
What is the resulting intensity pattern?

( Method I )

Reminder: N-slit interference:

\[
\langle I \rangle \propto \left[ \frac{\sin \left( \frac{N\delta}{2} \right)}{\sin \left( \frac{\delta}{2} \right)} \right]^2
\]

Where \( \delta \) is the phase difference between near-by slits: \( \delta = \frac{d\sin \theta}{\lambda}2\pi \)

Consider the limit:

\[
d \to 0 \quad N \to \infty \quad Nd = D
\]

\[
\Rightarrow \delta \to 0 \quad N\delta = \frac{D\sin \theta}{\lambda}2\pi
\]

\[
\langle I \rangle \propto \left[ \frac{\sin \left( \frac{N\delta}{2} \right)}{\sin \left( \frac{\delta}{2} \right)} \right]^2
\]

We can define:

\[
\beta = \frac{N\delta}{2} = \frac{\pi D\sin \theta}{\lambda}
\]

\[
\Rightarrow \langle I \rangle \propto \left[ \frac{\sin \beta}{\beta} \right]^2
\]

Here we also assume that the intensity of individual point source is proportional to \( N^{-2} \).
Another method described in Georgi’s book: Do an integration over all point-like sources to calculate the total electric field

\[ C(k_x, k_y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy f(x, y) e^{-ik \cdot \vec{r}(x, y)} \]

Where \( C \) is proportional to the total electric field. The integrals are over the unite area of the point source and \( f \) is the shape of the sources. This is the Fourier transform of \( f(x, y) \)

Let’s consider a single slit experiment

\[ f(x, y) = \begin{cases} 
1 & \text{if } -\frac{D}{2} \leq x \leq \frac{D}{2} \\
0 & \text{if } |x| > \frac{D}{2} 
\end{cases} \]

\[ C(k_x, k_y) = \frac{1}{4\pi^2} \int_{-D/2}^{D/2} e^{-ik_x x} dx \int_{-\infty}^{\infty} e^{-ik_y y} dy \]

\[ = \delta(k_y) \frac{1}{2\pi} e^{-ik_x x} \left[ e^{-ik_x D/2} - e^{+ik_x D/2} \right] \]

\[ = \delta(k_y) \frac{1}{2\pi} 2 \sin k_x D/2 \]

Therefore

\[ |\vec{E}| \propto C \propto \frac{\sin k_x D/2}{k_x} \]

\[ I \propto |C|^2 \propto \frac{\sin^2 k_x D/2}{k_x} \]

since \( \frac{x}{r} = \frac{k_x}{k} = \frac{k_x \lambda}{2\pi} = \sin \theta \)

\[ \Rightarrow k_x = \frac{2\pi \sin \theta}{\lambda} \]

\[ \Rightarrow I \propto \frac{\sin^2 \left( \frac{\pi D}{\lambda} \sin \theta \right)}{\left( \frac{\pi D}{\lambda} \sin \theta \right)^2} \]

Define \( \beta \equiv \frac{\pi D \sin \theta}{\lambda} \)

\[ \langle I \rangle \propto \left( \frac{\sin \beta}{\beta} \right)^2 \]

Same result as method I!
Observation:
(1) If we increase the size of the slit D:
⇒ the width decreases!
(2) Distance between peaks $\propto \lambda$
Principle Maximum $\Rightarrow$ The width is larger for red light (longer wave length) than blue light (shorter wave length).

(3) Intensity decreases quickly $\propto \frac{1}{\beta^2}$ as a function of $\beta$ (or $\sin \theta$) if $D$ is large. On the other hand: if $D$ is smaller, intensity decreases slower.

Coming back to the double-slit experiment: make it even more realistic: include the effect from finite slit width:
\[ I = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \left( \frac{\sin \frac{N\delta}{2}}{\sin \frac{\delta}{2}} \right)^2 \]

\[ \beta = \frac{\pi D}{\lambda} \sin \theta \quad \delta = kd \sin \theta = \frac{2\pi d \sin \theta}{\lambda} \]

Let’s consider a pin hole or aperture:

One can do the integration and we found that the intensity is:

\[ I(\theta) = I_0 \left( \frac{J_1(\beta)}{\beta} \right)^2 \]

Where \( J_1 \) is a Bessel function of the first kind:

Solve:

\[ J_1(x) = 0 \implies x \approx 3.83 \]

\[ \implies \beta = 3.83 = \frac{\pi D}{\lambda} \sin \theta \]

\[ \implies \sin \theta \approx 1.22 \frac{\lambda}{D} \]

And so the resolution of a pin hole:

\[ \sin \Delta \theta \approx \Delta \theta = 1.22 \frac{\lambda}{D} \]

Such that we can separate the two peaks! Human pupil is 2-4 mm when narrow and 3-8 mm when wide. Take visible light which is around 500 nm. \( D \sim 5 \) mm. Resolution:

\[ \sim 1.22 \frac{\lambda}{D} \sim 1.22 \frac{5 \cdot 10^{-7}}{5 \cdot 10^{-3}} \sim 1.22 \cdot 10^{-4} \]
iPhone 7: 401 ppi

\[
\Delta x \sim \frac{2.54 \text{cm}}{401} \sim 6.3 \times 10^{-3} \text{ cm} \\
\Delta \theta \sim \frac{\Delta x}{10 \text{ cm}} \sim 3 \times 10^{-4}
\]

The human eye can resolve it! Will you buy the iPhone x with 40,000 ppi? If Apple put 2,000 pixels in 6 cm ~ the limit.

We have learned single slit diffraction.

\[
I = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \quad \beta = \frac{\pi D \sin \theta}{\lambda}
\]

This means that a laser pointer is not merely producing a pencil beam.

Suppose \( \lambda = 500 \text{ nm} = 5 \times 10^{-7} \text{ m} \) and \( D = 1 \text{ mm} = 1 \times 10^{-3} \text{ m} \).

Opening angle:

\[
\theta \approx 1.22 \frac{\lambda}{D} = 6 \times 10^{-4}
\]

If we shoot a laser to moon: \( L = 4 \times 10^8 \text{ m} \) the radius of the principle maxima is 240 km!!