BARTON ZWIEBACH: De Broglie, as we discussed last time, we spoke about waves. Matter waves. Because people thought, anyway light is waves so the surprising thing that would be that matters are waves.

So a free particle with momentum p can be associated to a wave-- to a plane wave, in fact-- plane wave-- with wavelength lambda equals Planck's constant over p. So this wave is what eventually becomes a famous wave function. So de Broglie was writing the example or trying to write the example of what eventually would become wave functions, and the equations for this wave would become the Schrodinger equation. So really, this is a pillar of quantum mechanics. You're getting there when you talk about this wave.

So Schrodinger's equation is a wave equation for these matter waves and this plane wave eventually will become the wave function and there is a Schrodinger equation for it. So it's a wave of what? he was asking-- de Broglie had little idea what that wave was. When you have waves, like electromagnetic waves, you have polarization, you have directional properties, the electric field points in some direction, the wave is polarized. Is there a same property for the wave function? The answer is yes. We'll have to wait a little in 8.04 to see it, but it has to do with spin. When the particles have spin, there are directional properties of the wave and typically, you use several wave functions that correspond to directional components of this wave.

So photons are spin 1 particle electrons or spin 1/2 particles, so there will be directional properties to it. But to begin with, let's consider cases where this directional properties don't matter so much, and for the case of electrons, if the electrons have small velocities or they are inside small magnetic fields where some of these properties of the spin is important, we can ignore that and work with a wave function that will be a complex number. So it will be a wave function-- we'll denote it by the letter psi, capitol psi-- that depends on position and time, and that's the wave function. And to begin with, simplicity will be one of them, and it's a complex number. And it's just one wave function.

And the obvious questions about this wave function are, is it measurable and what it's meaning is? So is it measurable? And what is its meaning? But to understand some of that-- in fact, to get to realize that these waves are no ordinary waves, we're going to think a little about what it means to have a wave whose wavelength is inversely proportional to the momentum of a particle. That's certainly a strange statement and probably these are strange waves as you will see. And by understanding that these are strange waves, we are ready to admit later on that the interpretation could be somewhat surprising as well. And the nature of this number is, again, a little strange as you will see.

So all of that will come by just looking a little more in detail at this formula of de Broglie and asking a very simple question-- you have this particle moving with some momentum and I say, OK, it has this much wavelength. How about the person? If one of you is moving relative to me, like you usually do with Einstein, these observers that are boosted, but let's just do non-relativistic physics, what is called the Galilean transformation, in which there will be another observer moving with constant velocity with respect to you and you and that other observer compare the results on the momentum and the wavelength and see if you find a reasonable agreement or things make sense.

So we're going to try to think of p is h over lambda. And 2 pi's are very useful sometimes. So you put an h over 2 pi here and a 2 pi over lambda and you rewrite this in terms of quantities that are a little more common-- one is h-bar and the other is called the wave number k. So these are these two constants and this one is called the wave number. The 2 pi's are all over the place. If you have a wave with some frequency nu, there's also a frequency omega, which is 2 pi nu.

So we're going to look at this wave, and it has some momentum and some wave number, therefore it has some wavelength, and let's see-- if we compare things between two different frames, what do we find? So we'll put the frame S and a frame S prime moving with some velocity plus v in the x direction. So the setup is relatively common. We'll have one frame here that's the S frame, and it's the x-axis of the S frame. And the S prime frame coincided with the S frame at time equals 0-- now it's moving, so it's now over here, it's S prime. It has moved a distance of vt-- it's moving with velocity v and there's t. And S prime has-- and x is x prime.

On this, we're going to write a few things. We're going to say we have a particle of mass m. It has velocity v underbar, otherwise I'm going to get all my velocities confused. So this velocity v is the velocity of the frame, v underbar is the velocity of the particle, and v underbar prime, because the velocity depends on the frame of reference.

Similarly, it will have a momentum-- and all the things we're doing are nonrelativistic, so momentum p or p prime. Here is the particle. And that's the position x prime with the particle. And that's a position x of the particle. So that's our system. This particle is moving with some velocity over here, and we're going to compare these observations.

So it's simple to write equations to relate the coordinates. So x prime, for example, is the value of the corner at x of the particle minus the separation. So x minus vt. And I should say it here, we're assuming that t prime is equal to t, which is good nonrelativistically. It's fairly accurate. But that's the exact Galilean answer-- when you talk about Galilean transformations and Galilean physics, it's very useful. Even in condensed matter physics, people write these days lots of papers about Galilean physics, so when you have particles moving with low velocities, it's accurate enough, so might as well consider it.

And these are the two ways you transform coordinates, coordinates and time. So from this, we can take a time derivative talking about the particle-- so we have dx prime and dt prime or t, it's your choice-- I guess I should put dt prime here, dx/dt minus v, which means that the velocity v prime underbar is equal to v underbar minus little v. And that's what you expect. The difference of velocities is given by the subtraction of the velocity that the frame is moving. So if this particular has some high velocity with respect to the lab frame with respect to this frame, it will have a smaller velocity. So this sine seems right. And therefore, multiplying by m, you get that p prime is equal to p minus mv.

So if you have that, we would have that lambda prime, the de Broglie wavelength measured by either running person, is equal to h over p prime is equal to h over p minus mv, and it's quite different, quite substantially different from h over p, which is equal to the de Broglie wavelength seen in the lab. So these two de Broglie wavelengths will differ very substantially.

If this would be a familiar type of wave-- like a sound wave that propagates in the medium, any kind of wave that propagates in a medium, like a water wave or any wave of that type-- this would simply not happen. In the case of those waves, you get a Doppler shift-- omega is changed-- but the wavelength really doesn't change. The wavelength is almost like something you look at when you take a picture and whether you take a picture of the wave as you run or you take a picture of the wave as you are sitting still, you'll measure the same wavelength.

Let me convince you of that. It's an opportunity to just do a little more formal transformations, because these are going to be Galilean transformations, simple transformations. So our first observation is that the de Broglie wavelength don't agree, which pretty much, I think, intuitively is saying that if you could just sort of see those waves and measure the distance between peaks, they should agree, but they don't, so there's something very strange happening here.