

PROFESSOR: How about the expectation value of the Hamiltonian in a stationary state? You would imagine, somehow it has to do with energy eigenstates and energy. So let's see what happens. The expectation value of the Hamiltonian on this stationary state. That would be $\int dx \psi^*(x) \hat{H} \psi(x)$. And we're going to see this statement that we made a few minutes ago become clear. Well what do we get here? $\int dx \psi^*(x) e^{-iEt/\hbar} \hat{H} e^{iEt/\hbar} \psi(x)$. And \hat{H} couldn't care less about the time dependence, that exponential is irrelevant to \hat{H} .

That exponential of time can be moved across and cancelled with this one. And therefore you get that this is equal to $\int dx \psi^*(x) \hat{H} \psi(x)$, which is a nice thing to notice. The expectation value of H on the full stationary state is equal to the expectation value of H on the spatial part of the stationary state. That's neat. I think it should be noted. So it's equal to the H of little ψ of x . But this one, we can evaluate, because if we are in a stationary state, $\hat{H} \psi$ of x is E times ψ of x . So we get an $E \int dx \psi^* \psi$, which we already show that integral is equal to one, so we get the energy.

So two interesting things. The expectation value of this quantity of H in the stationary state is the same as its expectation value of H in the spatial part, and it's manually equal to the energy. By the way, you know, these states are energy eigenstates, these ψ of x 's, so you would expect zero uncertainty because they are energy eigenstates. So the zero uncertainty of the energy operator in an energy eigenstate. There's zero uncertainty even in the whole stationary state. If you have an H^2 here, it would give you an E^2 , and the expectation value of H is equal to E , so the expectation value of H^2 minus the expectation value of H squared would be zero. Each one would be equal to E^2 . Nothing would happen, no uncertainties whatsoever.

So let me say once more, in general, being so important here is the comment that the expectation value of any time independent operator, so comments 1, the expectation value of any time-independent operator Q in a stationary state is time-independent. So how does that go? It's the same thing. $\langle Q \rangle$ on the ψ of x and t is general, now it's $\int dx \psi^*(x, t) Q \psi(x, t)$ equals $\int dx \psi^*(x) Q \psi(x) e^{-iEt/\hbar} e^{iEt/\hbar}$. And I'll put the whole thing here. $\langle Q \rangle = \int dx \psi^*(x) Q \psi(x) e^{-iEt/\hbar} e^{iEt/\hbar}$.

So it's the same thing. Q doesn't care about time. So this factor just moves across and cancels this factor. The time dependence completely disappears. And in this case, we just get-- this is equal to $\int dx \psi^* Q \psi$, which is the expectation value of Q on ψ of x , which is clearly time-independent, because the state has no time anymore and the operator has no time. So everybody loves their time and we're in good shape.

The second problem is kind of a peculiarity, but it's important to emphasize superposition. It's always true, but the superposition of two stationary states is or is not a stationary state?

STUDENT: No.

PROFESSOR: No, good. It's not a stationary state in general because it's not factorizing. You have two stationary states with different energies, each one has its own exponential, and therefore, the whole state is not factorized between space and time. One time-dependence has one space-dependence plus another time-dependence and another space-dependence, you cannot factor it. So it's not just a plain fact. So the superposition of two stationary states of different energy is not stationary.

And it's more than just saying, OK, it's not stationary. What it means is that if you take the expectation value of a time-independent operator, it may have time-dependence, because you are not anymore guaranteed by the stationary state that the expectation value has no time-dependence. That's how, eventually, these things have time-dependence, because these things are not [INAUDIBLE] on stationary states. On stationary states, these things would have no time-dependence.

And that's important, because it would be very boring, quantum mechanics, if expectation values of operators were always time-independent. So what's happening? Whatever you measure never changes, nothing moves, nothing changes. And the way it's solved is because you do have those stationary states that will give you lots of solutions. And then we combine them. And as we combine them, we can get time-dependence and we can get the most [INAUDIBLE] equation.