8.08 Problem Set # 5

March 2, 2005
Due March 9, 2005

Problems:

1. At finite temperature, a semiconductor contains electrons and holes. An electron and a hole can annihilate and release an energy Δ:

\[ e + h \rightarrow \Delta \]

(You may assume each electron and each hole have an internal energy \( \Delta/2 \).) Here we assume the electrons and the holes have the same mass \( m \) and the temperature is \( T \).

(a) Find the densities of the electrons \( n_e \) and the holes \( n_h \) in a undoped semiconductor. (In a undoped semiconductor \( n_e = n_h \).)

(b) Find the densities of the electrons \( n_e \) and the holes \( n_h \) in a doped semiconductor. (In a doped semiconductor \( n_e - n_h = n_d \) where \( n_d \) is the density of doping which is fixed.)

2. If we roll two dices, we get a pair of random numbers \((n_1, n_2)\).

(a) Consider two random numbers

\[ k_+ = n_1 + n_2, \quad k_- = n_1 - n_2. \]

Are \( k_+ \) and \( k_- \) independent random numbers?

(b) Consider two random numbers

\[ m_+ = (n_1 + n_2) \mod 6, \quad m_- = n_1 \mod 6. \]

Are \( m_+ \) and \( m_- \) independent random numbers?

3. (a) A pendulum is formed by a mass \( M \) and string of length \( L \). Calculate the thermal fluctuations of the position of the mass: \( \Delta x = \sqrt{\langle (x - \bar{x})^2 \rangle} \). Assume the air temperature is \( T \).

(b) Calculate the value of \( \Delta x \) assuming \( M = 1 \text{g}, L = 10 \text{cm} \), and \( T = 300 \text{K} \).