8.08 Problem Set # 8

March 30, 2005
Due April 6, 2005

Problems:

1. Consider a gas of bosonic sodium atoms confined in a quadratic potential well \( U(r) = \frac{1}{2}m\omega_0^2 r^2 \) where \( m \) is the mass of the sodium atom. The characteristic length of the oscillator potential is \( r_0 = \sqrt{\hbar/m\omega_0} = 5 \times 10^{-3} \text{cm} \).

   (a) Ignore the interaction between the sodium atoms, find the size of the condensed sodium atoms at \( T = 0 \). How does the size of the condensation depends on the number of particles?

   (b) For interacting bosons, the shape of condensation at \( T = 0 \) is determined by

   \[
   \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial r^2} + (U(r) - \mu) + g|\psi(r)|^2 \right] \psi(r) = 0
   \]

   In Thomas-Fermi approximation, we assume the wave function \( \psi \) is smooth and drop the \( \partial^2_r \) term. In this case the shape of condensation is determined by

   \[
   [(U(r) - \mu) + g|\psi(r)|^2] \psi(r) = 0
   \]

   Now, how does the size of the condensation depends on the number of particles?

   (c) Fig. 15.2 of Huang’s book shows measured shapes of condensation. The maximum density is \( 10^{11} \text{cm}^{-3} \) for the shape near \( T = 0 \). Using the data provided by the curve, find the scattering length \( a \) of the sodium atom. (Note \( a \) and \( g \) is related through Eq. (15.3) in Huang’s book.)

2. Problem 15.9 in K. Huang’s book.