The Global Positioning System and Relativity: A <10% test of general relativity everyday

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Overview

• Original design of the Global Positioning System (GPS)
• Use by “non-authorized” users
• Selected applications
• Relativistic effects
GPS Original Design

- Started development in the late 1960s as NAVY/USAF project to replace Doppler positioning system
- Aim: Real-time positioning to < 10 meters, capable of being used on fast moving vehicles.
- Limit civilian ("non-authorized") users to 100 meter positioning.
GPS Design

• Innovations:
  – Use multiple satellites (originally 21, now ~28)
  – All satellites transmit at same frequency
  – Signals encoded with unique “bi-phase, quadrature code” generated by pseudo-random sequence (designated by PRN, PR number): Spread-spectrum transmission.
  – Dual frequency band transmission:
    • L1 ~1.575 GHz, L2 ~1.227 GHz
    • Corresponding wavelengths are 190 mm and 244 mm
Latest Block IIR satellite
(1,100 kg)
Measurements

• Measurements:
  – Time difference between signal transmission from satellite and its arrival at ground station (called “pseudo-range”, precise to 0.1–10 m)
  – Carrier phase difference between transmitter and receiver (precise to a few millimeters)
  – Doppler shift of received signal

• All measurements relative to “clocks” in ground receiver and satellites (potentially poses problems).
Measurement usage

- “Spread-spectrum” transmission: Multiple satellites can be measured at same time.
- Since measurements can be made at same time, ground receiver clock error can be determined (along with position).
- Signal: $V(t, \bar{x}) = V_o \sin[2 \left( ft \ k \cdot \bar{x} \right) C(t)]$
  
  - $C(t)$ is code of zeros and ones (binary).
  - Varies discretely at 1.023 or 10.23 MHz.
Measurements

• Since the C(t) code changes the sign of the signal, satellite can be only be detected if the code is known (PRN code)
• Multiple satellites can be separated by “correlating” with different codes (only the correct code will produce a signal)
• The time delay of the code is the pseudo-range measurement.
Position Determination (perfect clocks).

• Three satellites are needed for 3-D position with perfect clocks.

• Two satellites are OK if height is known)
Position determination: with clock errors: 2-D case

- Receiver clock is fast in this case, so all pseudo-ranges are short.
Positioning

• For pseudo-range to be used for positioning need:
  – Knowledge of errors in satellite clocks
  – Knowledge of positions of satellites

• This information is transmitted by satellite in “broadcast ephemeris”

• “Differential” positioning (DGPS) eliminates need for accurate satellite clock knowledge.
GPS security: SA

• To stop non-authorized users from getting the full accuracy of GPS, the military until May 2000 “corrupted” the GPS signals.
• Selective Availability (SA) “dithered” the clocks by time equivalent of ±100 meters.
• Turned off because ineffective
  (Example shown later)
GPS security: AS

• To ensure that military systems were not corrupted by false GPS transmission, Anti-spoofing (AS) is enabled on all satellites.
• At L1 frequency: GPS satellites use two C(t) sequences: Course Acquisition C/A code and Precise Positioning code (P code).
• P-code is modified under AS to the Y-code which only authorized users know.
Satellite constellation

• Since multiple satellites need to be seen at same time (four or more):
  – Many satellites (original 21 but now 28)
  – High altitude so that large portion of Earth can be seen (20,000 km altitude —MEO)
Current constellation

- Relative sizes correct (inertial space view)
- “Fuzzy” lines not due to orbit perturbations, but due to satellites being in 6-planes at 55° inclination.
Satellite Availability (smallest number above 15° minimum elevation)
Positioning accuracy

• Best position accuracy with pseudo-range is about 20 cm (differential) and about 5 meters point positioning.
• For many applications we want better accuracy
• For this we use “carrier phase” where “range” measurement noise is few millimeters
Carrier phase positioning

• To use carrier phase, need to make differential measurements between ground receivers.

• Simultaneous measurements allow phase errors in clocks to be removed i.e. the clock phase error is the same for two ground receivers observing a satellite at the same time (interferometric measurement).
Phase positioning

- Use of carrier phase measurements allows positioning with millimeter level accuracy and sub-millimeter if measurements are averaged for 24-hours.
Examples of positioning results

Pseudorange Point Positioning
Scatter 21 m and 23 m

Pseudorange Differential Positioning
Scatter 0.20 m and 0.15 m

Phase Differential Positioning
Scatter 0.008 m and 0.002 m

SA on
Summary

• Use of differential measurements with carrier phase allows very precise position determination (independent largely of security features).

• We use these measurements in Earth science for deformation studies and atmospheric studies
Tectonic Deformation Results

• “Fixed GPS” stations operate continuously and by determining their positions each day we can monitor their motions relative to a global coordinate system

• Temporary GPS sites can be deployed on well defined marks in the Earth and the motions of these sites can be monitored (campaign GPS)
Example of motions measured in Pacific/Asia region

- Fastest motions are >100 mm/yr
- Note convergence near Japan

Motion after Earthquakes. Example from Hector Mine, CA

Continued motion tells us about material characteristics and how stress is re-distributed after earthquake.
Relativistic effects

• General relativity affects GPS in three ways
  – Equations of motions of satellite
  – Rates at which clock run
  – Signal propagation
• In our GPS analysis we account for the second two items
• Orbits only integrated for 1-3 days and equation of motion term is considered small
Clock effects

- GPS is controlled by 10.23 MHz oscillators
- On the Earth’s surface these oscillators are set to $10.23 \times (1 - 4.4647 \times 10^{-10})$ MHz (39,000 ns/day rate difference)
- This offset accounts for the change in potential and average velocity once the satellite is launched.
- The first GPS satellites had a switch to turn this effect on. They were launched with “Newtonian” clocks
Propagation and clock effects

• Our theoretical delay calculations are made in an Earth centered, non-rotating frame using a “light-time” iteration i.e., the satellite position at transmit time is differenced from ground station position at receive time.
• Two corrections are then applied to this calculation
Corrections terms

• Propagation path curvature due to Earth’s potential (a few centimeters)

\[
\frac{2GM}{c^3} \ln \frac{R_r}{R_s} \frac{R_s}{R_r}
\]

• Clock effects due to changing potential

\[
\frac{\sqrt{GM}}{c^2} e^{\sqrt{a} \sin E}
\]

• For \( e=0.02 \) effect is 47 ns (14 m)
Effects of Selective Availability

PRN 03 (June 14)

Clock error (ns) vs Time (hrs)

- Clock SA (ns) 1999
- Clock NoSA (ns) 2000

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Relativistic Effects

PRN 03 Detrended; e=0.02

Clock error (ns)

Time (hrs)

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Tests of General Relativity

• In the parameterized post-Newtonian formulation, the time delay expression becomes:

\[
\frac{\sqrt{GM}}{c^2} \frac{(1 + \frac{1}{2})}{2} e^{\sqrt{a} \sin E}
\]

• In PPN, \( \gamma \) is the gravitational term. In general relativity \( \gamma = 1 \)

• The clock estimates from each GPS satellite allow daily estimates of
Using GPS to determine

- Each day we can fit a linear trend and once-per-revolution sin and cos terms to the each of the 27-28 GPS satellites.
- Comparison between the amplitude and phase (relative to sin(E)) allows and estimate of gamma to be obtained
- Quadrature estimates allows error bound to be assessed (cos(E) term)
- Problem: Once-per-orbit perturbations are common. However should not be proportional to eccentricity.
Initial “quick” results

Amplitude comparison only
Consistent with GR to <10%
Only 1 week of data: Data after May 2000 could be used.
Conclusions

• GPS dual-use technology: Applications in civilian world widespread
  – Geophysical studies (mm accuracy)
  – Engineering positioning (<cm in real-time)
  – Commercial positioning: cars, aircraft, boats (cm to m level in real-time)

• Relativistic effects are large but largely constant

• However due to varying potentials and velocities effects can be seen

• Some effects are incorporated by convention

• Need to keep in mind “negligible effects” as accuracy improves