8.251 – Homework 11

B. Zwiebach

Due Friday, May 11.

1. (15 points) Problem 13.6.

2. (10 points) Counting bosonic states.
   
   (a) Consider $k$ ordinary commuting oscillators $a^i$, with $i = 1, \ldots, k$. How many products of the form $a^{i_1}a^{i_2}$ can be built? How many $a^{i_1}a^{i_2}a^{i_3}$? How many $a^{i_1}a^{i_2}a^{i_3}a^{i_4}$?
   
   (b) List and count the states in the $\alpha' M^2 = 3$ level of the open bosonic string. Confirm that you get the same number of states predicted by the generating function.

3. (5 points) Generating function for the unoriented bosonic open string theory.
   
   Write a generating function for the unoriented bosonic open string theory by starting with the generating function for the full oriented theory and adding a term that implements the projection to unoriented states.

4. (10 points) Massive level in the open superstring.
   
   (a) Consider eight anticommuting variables $b^i$, with $i = 1, \ldots, 8$. Ignoring signs, how many inequivalent products of the form $b^{i_1}b^{i_2}$ can be built? How many $b^{i_1}b^{i_2}b^{i_3}$? How many $b^{i_1}b^{i_2}b^{i_3}b^{i_4}$?
   
   (b) Consider the first and second excited levels of the open superstring ($\alpha' M^2 = 1, 2$). List the states in the NS sector and the states in the R sector. Confirm that you get the same number of states.

5. (10 points) Closed string degeneracies.
   
   In closed string theories the value of $\frac{1}{2} \alpha' M^2$ is obtained by adding the (identical) contributions of $\alpha' M^2$ for the left and right sectors.
   
   (a) State the values of $\frac{1}{2} \alpha' M^2$ and give the degeneracies for the first five mass levels of the closed bosonic string theory.
   
   (b) State the values of $\frac{1}{2} \alpha' M^2$ and give the separate degeneracies of bosons and fermions for the first five mass levels of the type II closed superstrings.

6. (10 points) Problem 14.1

7. (10 points) Problem 14.2.

8. (10 points) Problem 14.3.