Quiz 1

Name __________________________ Solutions ____________________________

Last First (please print)

1. Closed book exam; you may use one page of notes.
2. Be sure to attempt all problems.
3. Important: Wherever possible, try to solve the problems using general analytic expressions. Plug in numbers only as a last step.

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Problem 1 (25 points)

A spectrum is taken of a single star (i.e., one not in a binary). Among the observed spectral lines is one from oxygen whose rest wavelength is 5007 Å. The Doppler shifted oxygen line from this star is observed to be at a wavelength of 5012 Å. The star is also observed to have a proper motion, μ, of 1 arc second per year (which corresponds to ~1.5 × 10^{-13} radians per second of time). It is located at a distance of 60 pc from the Earth.

(a - 19 pts) What is the 3 dimensional velocity of the star?

(b - 6 pts) What parallax would be measured for this star?

\[ U_t = \frac{\Delta \lambda}{\lambda} c = \frac{5}{5000} c = 300 \text{ km/s} \]

\[ U_T = 1.5 \times 10^{-13} \times D = \Theta D = \mu D = 277 \text{ km/s} \]

\[ \text{Speed} = 408 \text{ km/s} \]

(b) \[ \text{Parallax} = \frac{1}{60} \]
Problem 2 (25 points)

The velocity curve for a double-line spectroscopic binary is shown in the sketch. The system is viewed edge on, i.e., with an inclination angle of $i = 90^\circ$, so that the maximum possible Doppler shifts for this system are observed.

(a - 3 pts) Find the orbital period.
(b - 3 pts) Find the orbital speed of star 1 and star 2.
(c - 3 pts) What is the mass ratio of the two stars?
(d - 16 pts) Find the masses of the two stars, preferably in solar units.

\[ P = 5.5 - 1.5 = 4 \text{ days} \]
\[ v_1 = 300 \text{ km/s} \quad v_2 = 100 \text{ km/s} \quad (\text{since } i = 90^\circ, \sin i = 1) \]
\[ m_1 \, v_1 = m_2 \, v_2 \quad \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{100}{300} = \frac{1}{3} \quad \text{so, } m_1 < m_2 \]

\[ \frac{G \, m_1 \, m_2}{(r_1 + r_2)^2} = \frac{m_1 \, v_1^2}{r_1} \quad \text{Max. } F = m_1 \frac{G \, m_1 \, m_2}{r_1^2 (1 + \frac{m_1}{m_2})} = m_1 \, v_1^2 \]

\[ m_2 = \frac{v_1^2 \, r_1 \, (1 + \frac{m_1}{m_2})^2}{G} \quad \text{But } \frac{2 \pi \, r_1}{P} = v_1 \]

\[ m_2 = \frac{v_1^2 \, v_1 \, P \, (1 + \frac{m_1}{m_2})^2}{2 \pi \, G} = \frac{v_1^3 \, P \, (1 + \frac{1}{3})^2}{2 \pi \, G} \]

\[ m_2 = 3.96 \times 10^{34} \, g = 19.8 \, M_\odot \]

\[ m_1 = 1.39 \times 10^{34} \, g = 6.6 \, M_\odot \]
Problem 3 (25 points)

(a - 9 pts) A candle has a power in the visual band of ~3 Watts. When this candle is placed at a distance of 3 km it has the same apparent brightness as a certain star. Assume that this star has the same luminosity as the Sun in the visual band (~10^{26} Watts). How far away is the star?

\[ \text{Same flux } \Rightarrow \frac{3}{(3 \text{ km})^2} = \frac{10^{26}}{d^2} \]

\[ d = 10^{13} \times \frac{3}{\sqrt{3}} = 1.7 \times 10^{13} \text{ km} \]

\[ d = 0.56 \text{ pc} \]

(b - 10 pts) Two stars have the same surface temperature. Star 1 has a radius that is 2.5 times larger than the radius of star 2. Star 1 is ten times farther away than star 2. What is the difference in apparent magnitude between the two stars?

\[ m_1 = -2.5 \log \left[ \frac{T_1}{R_1} \right] + \text{const} \]

\[ m_2 = -2.5 \log \left[ \frac{T_2}{R_2} \right] + \text{const} \]

\[ m_1 - m_2 = -2.5 \log \left[ \frac{R_1}{R_2} \right] - 2.5 \log \left[ \frac{d_2}{d_1} \right] \]

\[ = -2.5 \log 2.5 + 5 \log 10 \]

\[ = -2 + 5 = 3 \]

\[ m_1 - m_2 = 3 \]

(c - 6 pts) A certain red giant has a radius that is 500 times that of the Sun, and a temperature that is 1/2 that of the Sun's temperature. Find its bolometric (total) luminosity in units of the bolometric luminosity of the Sun.

\[ L_{\text{bol}} = 4\pi^6 T_4^4 R_4^2 \]

\[ L_0 = 4\pi^6 T_0^4 R_0^2 \]

\[ L_{\text{bol}} = \left( \frac{T_4}{T_0} \right)^4 \left( \frac{R_4}{R_0} \right)^2 L_0 = \left( \frac{4}{2} \right)^4 \left( 500 \right)^2 = 1.56 \times 10^4 L_0 \]
Problem 4. (25 points)

Short answer questions:

(a - 9 pts) A radio interferometer, operating at a wavelength of 1 cm, consists of 100 small dishes, each 1 m in diameter, distributed randomly within a 1 km diameter circle. What is the angular resolution of a single dish? What is the angular resolution of the interferometer array for a source directly overhead?

\[ \theta_{\text{single}} \approx \frac{\lambda}{D_1} = \frac{1}{100} = 10^{-2} \text{ radians} \approx 0.6 \]

\[ \theta_{\text{array}} = \frac{\lambda}{D_{\text{array}}} = \frac{1}{10^5} = 10^{-5} \text{ radians} \approx 2'' \]

(b - 8 pts) Sketch the mirrors for one type of optical telescope and label what shapes their surfaces must have. Sketch a few light rays incident along the optical axis until the point where they reach the focal plane.

![Newtonian and Cassegrain Telescopes](image)

(c - 8 pts) Briefly describe and sketch how an X-ray telescope works?

"Double reflection from a combination paraboloid-hyperboloid, at grazing angles smaller than the critical angle for total external reflection."

![X-ray Telescope Schematic](image)