DYNAMICS OF HOMOGENEOUS EXPANSION

Poisson’s Equation:
\[ \nabla^2 \phi = 4\pi G \rho, \quad \text{where } \vec{g} = -\nabla \phi. \]

where \( \rho \) is the mass density, \( \nabla^2 \) is the Laplacian:
\[ \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \]

and \( \nabla \) is the gradient:
\[ \nabla \equiv \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}. \]

\[ \vec{g} = -\frac{GM}{r^2} \hat{r} \implies \oint \vec{g} \cdot d\vec{a} = -4\pi GM_{\text{enclosed}} \]
Newton argued that there could be no acceleration, because there is no preferred direction for it to point. Complication: acceleration is measured relative to an inertial frame, which Newton defined as the frame of the “fixed stars”. But if the universe collapses, then there are no fixed stars. In the absence of an inertial frame, all accelerations, like velocities, are relative. When all accelerations are relative, any observer can consider herself to be non-accelerating. She would then see all other objects accelerating radially toward herself. Like the velocities of Hubble expansion, this picture looks like it has a unique center, but really it is homogenous.
Mathematical Model

\[ \mathbf{v}_i = H_i \mathbf{r} \]

\[ R_{\text{max},i} \]