\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G \left( \frac{\rho_{m,0} + \rho_{\text{rad},0} + \rho_{\text{vac}}}{a^3(t)} \right) - \frac{k c^2}{a^2}.
\]

Define

\[
\Omega_{k,0} \equiv -\frac{k c^2}{a^2(t_0) H_0^2}.
\]

Finally,

\[
\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{\text{rad},0} - \Omega_{\text{vac},0}.
\]

\[
t_0 = \frac{1}{H_0^2} \int_0^1 \frac{xdx}{\sqrt{\Omega_{m,0}x + \Omega_{\text{rad},0} + \Omega_{\text{vac},0}x^4 + \Omega_{k,0}x^2}}.
\]
fraction = \frac{\text{area of detector}}{\text{area of sphere}} = \frac{A}{4\pi \tilde{a}^2(t_0) \sin^2 \psi_D}.

Power suppressed by two factors of \((1 + z_S)\): one for redshift of photons, one for rate of arrival of photons.

\[
J = \frac{P_{\text{received}}}{A} = \frac{P}{4\pi (1 + z_S)^2 \tilde{a}^2(t_0) \sin^2 \psi_D},
\]

\[\tilde{a}(t_0) = \frac{cH_0^{-1}}{\sqrt{-\Omega_{k,0}}} \cdot \]

\[
0 = -c^2 \frac{dt^2}{\tilde{a}^2(t)} \frac{d\psi^2}{(1 + z_S)^2} \Rightarrow \frac{d\psi}{dt} = \frac{c}{\tilde{a}(t)}.
\]

\[
\psi(z_S) = \int_{t_S}^{t_0} \frac{c}{\tilde{a}(t)} \frac{dt}{(1 + z_S)^2}.
\]

\[
\psi(z_S) = \frac{1}{\tilde{a}(t_0)} \int_0^{z_S} \frac{c}{H(z)} \, dz.
\]

\[
\psi(z_S) = \sqrt{|\Omega_{k,0}|} \times \int_0^{z_S} \frac{dz}{\sqrt{\Omega_{m,0}(1 + z)^4 + \Omega_{\text{rad},0}(1 + z)^4 + \Omega_{\text{vac},0} + \Omega_{k,0}(1 + z)^4}}.
\]