Physics 8.321, Fall 2002
Homework #10

Due Monday, November 25 by 4:30 PM in the 8.321 homework box in 4-339B.

1. Sakurai: Problem 12, Chapter 3 (page 243)

2. Sakurai: Problem 15, Chapter 3 (page 244)

3. Sakurai: Problem 17, Chapter 3 (page 244)

4. Consider a magnetic monopole with magnetic charge $g$ located at the point $(0,0,d)$, and a second monopole with magnetic charge $-g$ located at the point $(0,0,-d)$. Find an expression for the vector potential associated with this configuration which is nonsingular for $r > d$. Describe the “Dirac string” of this configuration.

5. Compute the spherical harmonics $Y_{2m}(\theta, \phi)$ explicitly. Express these both as functions of $\theta, \phi$ and of $z = \cos \theta, x = \sin \theta \cos \phi, y = \sin \theta \sin \phi$. Compute $\sum_{m} |Y_{2m}|^2$.

6. An $l = 1$ state has $m = 1$. The state is rotated by an angle $\theta$ about the $y$ axis. What is the probability that a measurement of $L_z$ will yield $m = 1$? Repeat with a rotation by $\theta$ about the $z$ direction.

7. Using separation of variables, show that the eigenstates of the Hamiltonian for a spherically symmetric potential $V(r)$ may be written in the form

$$\Psi_{E,l,m} = R_{El}(r) Y_{lm}(\theta, \phi)$$

where $R_{El}(r) = \frac{1}{r} u_{El}(r)$ and $u_{El}$ satisfies

$$\left[ -\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + \frac{\hbar^2 l(l + 1)}{2mr^2} + V(r) \right] u_{El}(r) = E u_{El}(r).$$

8. Consider a particle in the 2D potential $(m = 1)$

$$V(x, y) = \frac{1}{2} \omega^2 (x^2 + y^2).$$

Use raising and lowering operators $a_x^+, a_y^+, a_x, a_y$ to compute the spectrum and degeneracies of the Hamiltonian. For each value of the energy, what eigenvalues of $J_z$ are possible? Are $H, J_z$ a complete set of commuting observables?