Midterm Quiz

Note: Some parts of problems can be solved without solving the earlier parts.

1. Magnetic moments and g-factors (18 points)
a. (4 p.) What is the magnetic moment of an atom in the following states: $^2S_{1/2}$, $^3S_1$, $^1D_2$, $^3P_0$?
   (give the values without derivation)
b. You have two negatively charged particles with spins $\vec{s}_1$ and magnetic moment $\mu_k = -g_k \mu_B \vec{s}_k / \hbar$
   The two spins couple to a total spin $\vec{S} = \vec{s}_1 + \vec{s}_2$. For a weak external magnetic field $B$ along the z-axis, find the possible values of the energy levels. Express the g-factor of the combined system by the individual g-factors $g_i$ and the spin quantum numbers for the following situations:
   (2 p.) $g_1 = g_2$
   (6 p.) $g_1 = -g_2$
   (6 p.) $g_1$ finite, $g_2 = 0$
   Although you could solve the problem in general and then specialize to the three cases, it is recommended to do the simple calculations separately --- otherwise you may lose partial credit for the easier parts.

2. Spontaneous emission (13 points)
a. (2 p.) For weak monochromatic resonant excitation with Rabi frequency $\omega_R$ of an atom with natural linewidth $\Gamma$, what is the excitation rate (=rate of photon absorption) using lowest order perturbation theory?
   (Note: The matrix element is $\hbar \omega_R / 2$; you don’t need to include numerical prefactors).
b. (4 p.) Using the result in a, explain how the excitation rate for a two-level system depends on the matrix element. Is this still true for broadband excitation?
c. (2 p.) In the limit of a very high (i.e. infinite) intensity, what is the rate of excitation?
   (give result without derivation). What is the cross section for photon absorption in this situation (assume monochromatic excitation)?
d. (5 p.) Specify the dominant multipole (such as E1 (electric dipole), E2, E3, ..., M1, M2, M3 ...) for spontaneous photon emission by an excited atomic electron in each of the following transitions. Assume simple hydrogenic wavefunctions without any relativistic or other corrections.
   $2p_{1/2} \rightarrow 1s_{1/2}$
   $2s_{1/2} \rightarrow 1s_{1/2}$
   $3d_{3/2} \rightarrow 2s_{1/2}$
   $2p_{3/2} \rightarrow 2p_{1/2}$
   $3d_{3/2} \rightarrow 2p_{1/2}$
3. Hydrogen like atoms (17 points)

Consider an atom which consists of a lithium nucleus (Z=3) and (instead of an electron) a negative muon $\mu$ with a mass $m_\mu/m_e=207$.

a. (4 p.) Derive (using any model you want) the expression for the Bohr radius for the hydrogen atom. (you can ignore numerical factors)

b. (3 p.) For the muonic atom above, what is Bohr radius and the binding energy of the 1s state, relative to the hydrogen atom (neglect all nuclear mass corrections).

c. (3 p.) Compare atoms with the same nuclear charge, but with an electron or a muon in the 1s state. What is the ratio of the relativistic corrections to the binding energy?

d. (4 p.) Consider a lithium nucleus (Z=3) with three electrons? In units of the Rydberg constant, what is the ground state energy? What is the ground state energy, if one of the electrons is replaced by a negative muon? Neglect all interactions between electrons and muons.

e. (3 p.) Mention at least three phenomena which affect the energy of only s-electrons?