Professor: Good afternoon. So in the last week of this semester, we will be finishing up the chapter on coherence. What we want to continue to explore today is the presence of a dark state. If you have a three level system, and that’s what we went through last week, we will always find one state which is dark, which means we fully illuminate the atom with laser light, but there is one state which is not coupled to the excited state which does not scatter light, and therefore is dark.

I showed you that for very general conditions, if you have two laser beams exciting the two states, there will always be a coherent superposition of the two states, which is the dark state. The dark state is the novel feature of three level systems, and I want to show you different perspectives of it. We started out by talking about just the existence of the dark state.

Then we talked about dark state transfer. You can regard, for detuned light, the dark state as the lowest energy state of the system. And then an adiabatic theorem tells you that you can keep particles in the dark state even if you change the laser parameters and change what state is now the dark state. This is the basis of coherent population transfer, or the famous STIRAP method.

There is another aspect of the dark state which gives us a possibility of lasing without inversion. As I reminded you, lasing has a threshold, which is inversion that you need more atoms in the excited state than in the ground state because the atoms in the ground state absorb the laser light and you want a net gain. However, if you have a dark state, you have a situation where the atoms do not absorb the laser light, and therefore, the conditions for net gain have changed. And indeed, lasing without inversion becomes possible.

I started to explain to you the way how lasing without inversion could come about. It relies on the fact that in a three level system, absorption destructively interferes but stimulated emission does not destructively interfere. And therefore, you can have lasing without inversion in such three level systems.
I showed you one possible realization, and this is about hydrogen in a DC electric field. If we mix the 2S and 2P states with an electric field, we have this three level structure. And for a laser tuned right in the middle between the two states, the two amplitudes for excitation cancel exactly, and therefore, you have a zero absorption feature. However, if you put now a little bit of population into, let's say, the upper state, this upper state in the wings of its profile has still gained for stimulated emission, and what we get is lasing without inversion.

Now you say, OK, but that's a hydrogen atom. Which atom is really degenerate between two levels, and you can split regeneracy with a static electric field. Well, you're already an expert at this point in the course. If the 2S and 2P state are widely separated, well, you add a photon, and the photon, which is in resonance with the 2S and 2P state, creates, in the dressed atom picture, degeneracy. Maybe I should have shown the P state higher. The 2S state with one more photon and the 2P state with one photon less have the same energy, and then you create exactly this situation.

So therefore, the way you can realize that in atoms other than hydrogen is use an AC electric field to mix S and P states. And I'll show you in five minutes a little bit more in detail what I mean by that.

There is a trivial realization which I want to mention for lasing without inversion, and this would be if you have a three level system with an excited state, two levels, g and f in the ground state. You may have an inversion for the e to g transition. And therefore, you can get lasing because the population in this state f is not coupled.

This is pretty trivial, but the more subtle part, of course, is that we can realize it using a driven system using a control laser by creating the same situation with the bright and the dark state, and population in the dark state is hidden from the light and does not absorb light. Let me just indicate that in both those states, we would have no absorption, dark state.

Those examples may raise the question whether whenever you have lasing without inversion, if you can find a basis where you have inversion again between the two levels which are relevant and the extra population is just hidden. I want to make two comments about it. This question is sometimes discussed in the literature, sometimes in a semi-controversial way. There are two comments about it.

One is when you start dressing up your system with laser beams, you have strong control lasers. You have two lasers, omega 1, omega 2. One is often a strong control laser and the
other one is the weak laser where we want to have lasing. You have actually a time dependent system driven by time dependent fields, and once you have a time dependent system, it's no longer clear what the eigenstates are, what the populations are, and what are the coherences. There is no longer a unique way to distinguish what are the eigenstates because every state is, so to speak, time dependent almost by definition.

On the other hand, I think the example I gave you with atomic hydrogen where you just a little bit of mixing with an electric field is an example where you genuinely have less population in the excited state than you have in the ground state, and at least the equation tells you, even without inversion, you have a net gain. So my own understanding of that situation is that in many situations, you can actually reduce it to a simple picture where you have simply hidden population in a dark state, but without any sort of unnatural definitions, you may not find that in some other systems. Questions about lasing without inversion? Nancy?

**STUDENT:** Is lasing without inversion important in lab, as opposed to lasing with inversion, or is it more about teaching us [INAUDIBLE]?

**PROFESSOR:** Well, lasing without inversion has definitely been touted as a way to get lasing deeper in the UV, to get lasing for very blue transitions because when you want to create inversion-- this actually has been the problem in creating x-ray lasers in atomic systems. If you have larger and larger energy separation, spontaneous emission scales with omega [? cube. ?] And so therefore, it becomes harder and harder to fulfill the ordinary gain equation.

**STUDENT:** So in those cases, even these last two methods [INAUDIBLE] because [INAUDIBLE]?

**PROFESSOR:** Well, lasing without inversion alleviates the requirement to build a laser, and so people have discussed that where it's really hard to create a laser in the conventional way, deep, deep in the blue of the UV or in the x-ray regime, that lasing without inversion may help. I'm not aware that any practical development has emerged from that because there is a price to be paid, and that is usually in the form of coherence. You need a certain degree of coherence in your system to be able to do that. It's an idea which is powerful, but as far as I know, there was no killer application of it.

The importance of dark states is definitely in slow light, manipulation, quantum computation, and concepts of storing light, and this is what we want to discuss next. Actually not next, but after next. What I first want to discuss is another aspect of the dark state, and this goes by the name EIT, electromagnetically induced transparency.
I can introduce this topic by a question to Radio Yerevan, is it possible to send a laser beam through a brick wall? And the answer of Radio Yerevan, if you know the joke, is always, in principle yes, but you need another very powerful laser. So in an incoherent way, of course, a very powerful laser can drill a hole into the wall and then the next laser can go through the wall without absorption, but you can be smarter. If the very powerful laser through coherence puts all the atoms in the brick in the beam pass of the laser into a coherent superposition state, and then they become a dark state, then your laser can go through a brick wall.

So can a laser beam penetrate an optically thick medium? And the answer is yes, with the help of another laser. Original ideas along those lines were formulated by Steve Harris, who has really pioneered this field, and he first considered special auto-ionizing excited states which couple to-- there were two pathways of coupling into the continuum, but later work has shown that it can be realized in a lambda system.

Let me talk about this conceptionally simpler realization in a three level lambda system. Let's assume we have again our normal three level system gf and an excited state which has a width, gamma. And we want to send a probe laser through a dense medium, and it would be completely absorbed by the resonance to the excited state.

But now we can have a strong coupling laser with a rapid frequency, omega c, and so if we drive the system very strongly, we can create a situation where the coupling laser does, if it's strong enough, complete mixing between the excited state and the ground state, and that means if you have two levels which are completely mixed, they are split by the energy or the frequency of the coupling.

So in other words, what we obtain is we have now two states, e plus f and e minus f. You know the how you should read is the following. You can just assume for a second the state f will degenerate with gamma and then I put in a very strong mix in between the two. This is exactly the example we had with hydrogen in an electric field, and then we get two states which have both width gamma over 2. They are strongly mixed, and the splitting is nothing else than the matrix element of the electric field.

But if you don't have two degenerate states and we add a photon, then the photon-- I mentioned it again and again in addressed atom picture-- creates a degeneracy. You can just add a photon, draw a dashed line, and this is your virtual state if you want. Or if you look at
Schrodinger’s equation, you have something which oscillates at the frequency of the state, f, but now you multiply it with an electric field which oscillates at the resonance frequency, and then you have something which oscillates at the sum of the two frequencies, and this is exactly what I indicated with a dashed line. So that’s how you create, so to speak, a degeneracy by using the frequency of the laser to overcome the energy splitting, and the result is that you have created exactly this excited state level structure.

And if you now look at the ground state, and our photon is tuned right between those two continua, then we have a dark resonance. And in order to accomplish this, I’m not going into any calculations here, but you need a sufficiently strong coupling laser who can accomplish that. For instance, coupling laser which is stronger than the spontaneous emission rate or decoherence rate gamma in the excited state.

So if we now scan the probe laser and we look for transmission, let me assume for simplicity that there is no relaxation between the two ground states. If the coupling laser strength is 0, then we have a broad feature, which is simply the single photon absorption of the probe light. If we have an infinitesimal coupling laser strength, we get a very, very sharp feature, and if the coupling laser is stronger and stronger, we get a window of transparency, and the width is given. The width delta omega of this central feature is given by the Rabi frequency of the coupling laser.

So what are possible applications? One is you can design non-linear materials. Usually, when you want to have frequency conversion processes or optical switches at one laser beam affects another laser beam, you want a very strong non-linear response of your medium, you usually get that if you go near resonance, but near resonance, you have strong absorption. But now, using the concept of EIT, you can have both. You can have the strong non-linear response of your medium near resonance but you suppress absorption, you get a window of transparency using EIT. So you can have near resonant materials without absorption.

Again, in principle, I’m not aware that this has really taken off in a bigger time. One would be if you want to do very sensitive spectroscopy. Assume you want to measure one isotope which has a tiny little abundance but you have to observe it against the background of a very strong isotope. If you could switch off the absorption of the background, the absorption of the strong isotope, you could still see a small amount of a trace isotope in the presence of a strong absorption line. So sensitive detection of trace elements. Questions about that?
I've given you qualitative pictures for lasing without inversion and for EIT. I want to give it a little bit more quantitative touch, not by going through to the optical Bloch equations which would be necessary to describe all features of it, but at least I want to give you one picture where you can derive and discuss things in a more quantitative way, and this is the eigenstate picture.

I've also done here what I've said several times, that when we have splittings between the levels, we can actually focus on what really matters, namely the detunings, by absorbing the laser frequency into the definition of our levels, and that's what I've done here. Instead of using levels g, f, and e, I have levels with g, f, e, but this is the photon number in laser field one and the photon number in laser field two. So if all of the photons, the laser field one and laser field two, would be resonance with their respective transitions, then all those three levels would be degenerate.

But now in the three level system, they're not degenerate only because we have a relative detuning delta, detuning small delta from the Raman resonance, and we have a detuning peak delta, which is sort the common detuning of the Raman laser from the excited state levels. So therefore, if I did define the Rabi frequencies, the Rabi frequencies over 2 are gain coefficient, and the Rabi frequencies are proportional to the electric field. Therefore, they scale with the photon numbers n and m in the two laser beams.

If I do that, I have a really very simple Hamiltonian. On the diagonal, we simply have the detuning of both laser beams form the excited state. Here we have the Raman detuning, and we have two couplings to the excited state. One is laser field one, g1, with n photons, and the other one is laser field g2 with m photons. Any questions about? Just setting up the simple equations which we have done a few times.

Let me focus first on the simple case that everything is on resonance. Then, if everything is on resonance, we have the structure which I've shown here. You can sort of say you have three levels, which are degenerate without any Rabi frequency, because the detunings are all zero, or all the diagonal is zero. And then the off diagonal matrix elements, the Rabi frequencies, are just spreading the three levels apart, and the general structure of this matrix is that in the middle, you always have a state which is just a superposition of g and f. So it's a dark state. It has no contribution in the excited state.

And the two outer states have equal contribution of the excited state. So the excited state has
been distributed over the two outer states, and the widths of those levels is therefore gamma
over 2. And we know when we had two levels and we were driving them with a Rabi frequency
resonantly, we had splittings which were just given by the Rabi frequency, and now the
splitting between the outer level and the dark state is the quadrature sum of the two Rabi
frequencies.

**PROFESSOR:**

So that's a very general structure.

I want to go back to the situation where one laser is a probe laser. Let's assume this is our
laser beam which wants to go through the brick wall, and the other laser has to prepare the
system. Let me discuss the limit where the photon number becomes very small in laser one,
the photon number n goes to unity, and this is much smaller than m. So we have the situation
of a weak probe field. If we have this limit, then the dark state has much, much more
amplitude in state g, and the state g is almost decoupled. It's in a trivial way. The dark state
and the laser beam is very strongly mixing the state f and the excited state.

So in this limit, we have a nice physical situation. I should actually point out that you can solve
most of those situations for three level systems analytically. It's just those explanations get
long and are not very transparent. So what I'm trying here in the classroom, I try to pick certain
examples-- weak probe field, resonance-- where we can easily understand the new features
which happen in the system.

So the situation we have now prepared is we have our dark state, which is level g, we said we
have only one photon in our probe beam, and we have lots and lots of photons in the coupling
laser which couples the other ground state, f, to the excited state, e. So we have the structure
that we have now two states which have half of the widths of the excited state, and they are
both bright. We can call one the bright state 1 and the other the bright state 2. The splitting is
related to the Rabi frequency. Let me just call it delta bar.

This is now the level structure which we have, and what I want to now emphasize in this
picture is the phenomenon of interference. We've talked about interference of amplitudes, but
now I want to take the system and show you how we get now interference when we send one
probe photon through the system. What we can now formulate is a scattering problem.

We have one photon in our probe beam in a special mode, but then, all the other modes are
unoccupied. And when we are asking, does the probe photon get absorbed? Does the weak
laser beam get stuck in the brick wall? We are actually are asking if it is possible that we
scatter the photon out of the mode and it gets absorbed. But you know absorption is actually always a two photon process involves spontaneous emission, and we have emitted the photon into another mode.

The scattering problem is a two photon process, and the matrix element needs an intermediate state, but now we have two. We start with one photon, we have the light atom coupling, we can go through bright state 1. From bright state 1, we have the light atom coupling again, and eventually, we go back to the ground state without four photon.

And here we have a detuning. Let's assume we are halfway detuned between the two bright states. And then we have a second amplitude, and it's indistinguishable. We have a Feynman double slit experiment, and everything here is the same except that we scatter through bright state 2. This matrix element, when it vanishes, this is now the condition of electromagnetically induced transparency.

But we want to now understand what happens when we detune the probe laser. So we have set up the system with a strong control laser, we have completely mixed the ground state with the excited state, and now we want to ask, can a weak probe laser go through the brick wall? How much of the probe laser is absorbed? So what we want to understand now, and this is a new feature I want to discuss now, what happens when we detune the probe laser by delta?

Well, it's clear, and I just wanted to show you the formula. We had two detunings here, and if we detune the probe laser by delta, that would mean now that those denominators are no longer opposite but equal. In one case, we add delta. In the other case, we subtract delta. And therefore, we have no longer the cancellation of the two amplitudes and we have the scattering of the photon.

This is sort of the framework, and I want to show you now several examples. I want to show you examples of a probe absorption spectra. It's more a little bit of show and tell. I want to show you the result if you would work that out. It's pretty interesting.

So first, I want to discuss the case where the coupling laser is really in resonance. We are near the one photon resonance. Let me assume first that the Rabi frequency of the coupling laser is much, much larger than gamma.

Then if you look at the probe transmission, we have the situation that we know when we are
right here in the middle, we have a window of transparency, but if we detune by the Rabi frequency over two, we hit the bright state 1, and if we detune in the opposite, we hit the bright state 2. And the splitting in this situation, delta bar, is nothing else than the Rabi frequency omega 2.

So what we have is we have the detuning in between, and we know already here is our special point, and that's what we have discussed for long, electromagnetically induced transparency. The two bright states are at plus and minus half the Rabi frequency of the coupling laser. We know that we have very strong absorption here. What you get is sort of broad feature.

But this was a situation when we drive the system very strongly. I now want to discuss the case that the coupling laser is much weaker than gamma. Then the splitting between the two bright states was the Rabi frequency, but the width is gamma, so then the two bright states pretty much merge into one continuum feature of width gamma.

For that situation, we have a broad feature of absorption, which is on the order of gamma, or if you have an opaque medium, of course, you put the absorption coefficient into the exponent of an exponential function and you get a blackout which is wider than gamma. But then we have our phenomenon of EIT, but the width of this feature is now much smaller than gamma.

So what we have here in those two situations where the lasers are close to resonance with the excited level, we have the situation that the strong absorption feature due to the bright states, either one strong feature of width gamma or two features here, those broad, similar photon absorption features, they really overlap with our window of transparency. And what I find very insightful is now to discuss the situation where we separate the two, and I want to show you what happens. It gives a very interesting profile.

So what we want to discuss now are the famous Fano profiles, and I want to discuss two photon absorption features. We have discussed the case where the one photon detuning. We want to go now to a large one photon detuning delta 2, and the new feature now is that this will separate the window of electromagnetically induced transparency from the broad absorption features.

Let me just draw a diagram of the states. As usual, we have our two states in a lambda transition, g and f. We have this continuum of the excited state, but now, and this is what often is done in the experiment, you're not using Raman lasers which are in resonance with the excited state. You're using Raman lasers-- here is the strong coupling lasers-- which are far
detuned. So here we have a detuning for laser two called gamma 2. The Rabi frequency is omega 2. Here we have a weak probe laser omega 1, and the detuning. Let's call it gamma 1.

So in order to keep the situation simple, we use a weak probe laser. Omega 1 is small. And our Raman detuning delta is the difference between the two single photons detuning, capital delta 2 minus delta 1. For these situations, there are nice analytic expressions, and together with the class notes, I will post a wonderfully clear paper by [? Lunis ?] and [INAUDIBLE] where the two authors discuss this situation.

Let's just figure out what are the features in the system. Let's just go through different situations. One is if the Raman detuning is zero, we should always get the phenomenon of electromagnetically induced transparency. If we don't have any coupling, then we simply have a two level system, and if we tune omega 1 into resonance, we get simple single photon absorption.

So if we look at the system of the three levels and we are asking what are now the relevant processes, one limit is, of course, the trivial limit that we have single photon absorption. This is, of course, a trivial case. Let's now go to the more interesting case that we have a coupling laser.

What happens now? We have our two states. The excited state is coupled with the laser and Rabi frequency omega 2. Now we know that the laser omega 2, if it is not on resonance, will give us an AC Stark shift. This AC Stark shift, we've gone through that several times. It's the matrix element or Rabi frequency squared divided by the detuning.

If we now bring in the probe laser, what are the features we expect? Well, there are two features. One is we've just discussed the trivial case above. If we tune the probe laser into resonance with the excited state, we have single photon absorption. We get a broad feature. It's almost like in the two level system. But in contrast to the case I just discussed above, the excited state level e has now an AC Stark shift so the resonance is shifted.

That's now becoming a four photon process because we need two photons going up and down with a coupling laser to create the AC Stark shift. And now we have a laser from the probe beam and the photon is scattered, so it's a four photon process and it will give us a broad resonance which is now AC stack shifted. But in addition, we have a resonance, which is the Raman resonance. When the Raman detuning is zero, then we absorb from the probe laser and we emit in a stimulated way with the coupling laser, and we have a stimulated two
Now, what is the width of this stimulated two photon transition? Well, we go from a stable ground state to-- I wanted to say another stable ground state, but this other stable ground state is now scattering photons. So because of the presence of a strong coupling laser, you have broadened this level f by photon scattering. You interrupt the coherent time evolution by scattering photons, and the photon scattering happens in perturbation theory by the amplitude to be in the excited state squared times gamma.

The scattering rate, gamma scattering, is Rabi frequency divided by detuning. This is the amplitude to be in the excited state. We square that, and then we multiply with gamma, and if I can trust my notes, it's a factor of two, which I don't want to discuss further. This is a quantitative argument. The analytic expressions are in the reference I've given to you.

So the situation which we have right now can be in a very powerful way summarized as follows. We have our ground state, we have two continua we can couple to. We can couple to the excited state which has a width, gamma, through a single photon, but there is the AC Stark shift. Or we can couple through a two photon Raman transition to the state f, but the width is much, much smaller because it is only the scattering rate due to the off resonant coupling laser.

Let me just write that down because I said a lot of things. So g couples now. When we detune the probe laser, we can be in resonance with this feature and we can be in resonance with that feature to a narrow and wide excited state. One excited state, of course, is the state f, but the coupling laser puts some character of the excited states into the state f.

And the most important thing now is the following. This is the theme I've emphasized again and again when we discussed three level systems. Those two states have a width, and the width means they spontaneously emit light, but they emit light into the same continuum. So if you start in the ground state, your probe laser has a photon and the photon gets scattered. You do not know through which channel it has been scattered.

So in general, and this is as far as I want to push it in this class, in three level system, we have now the interference between those two continua. One is narrow and one is wide. Let me just write that down because this is important, but both excited states couple to the same continuum. And by continuum, I mean the vacuum of all empty states where photons can be
scattered, and this is the condition for interference. And this is, of course, what gives rise to electromagnetically induced transparency.

I'll take your questions, but let me first give you a drawing which may illustrate, or summarize, what I've just said. So what we try to understand is what happens when we detune the probe laser. Until now, we had always the EIT feature, the EIT window, was completely overlapping with the one photon resonance, but now, because the coupling laser has a detuning of gamma 2, we have to use with the probe laser.

I have to trace back how the detunings are defined. The Raman detuning was big delta 2 minus big delta 1. If the detuning delta is chosen to be delta 2, we have the simple situation that we go from the ground state right to the excited state, and we have the feature which has a width of gamma. This is what you would call the single photon resonance. It is the single photon resonance.

It is due by resonantly coupling into this continuum, and the only feature of the coupling laser is that there's an AC Stark shift to it. Single normal resonance, almost like in a two level system, but the only addition is the AC Stark effect.

Now we have a second feature, which can be very sharp. This is when we do the two photon Raman process into the other ground state. Due to photon scattering, this resonance has a width of gamma scattering, and the position is not at zero at the naked Raman resonance. It also has an AC Stark shift because the coupling laser does an AC Stark shift to both the excited and the ground state. The coupling laser, the AC Stark shift, pushes ground and excited state in opposite directions.

So therefore, we find that at the Rabi frequency of the coupling laser divided by gamma over 2. The name of this feature is the two photon Raman resonance plus the AC Stark shift, which actually means it's a four photon process. The question is now, where is the electromagnetically induced transparency? We have introduced now specifically the absorption feature. We have identified a one photon absorption feature plus AC Stark shift, a two photon absorption feature plus AC Stark shift, but where is electromagnetically introduced transparency?

Here. Electromagnetically induced transparency is always at delta equals 0. You have to fulfill the Raman resonance, and this resonance is not affected by any AC Stark shift. It's always at
delta equals 0. So therefore, what that means is you have two absorption features, and you would think these are two Lorentzians, but they interfere and they go to exactly 0 at delta equals 0, and this is our EIT feature.

So there is an interference effect between a broad feature and a narrow feature, and this is found in many different parts of spectroscopy, but you also find it in nuclear physics whenever you have a narrow feature embedded into continuum. What we have here is a narrow feature and a broader feature, but a narrow feature and something which is broader or continuum is called a Fano resonance.

You actually have the same situation when you look at scattering of atoms. Many of you are familiar with Feshbach resonances. Well, a lot of people call it Fano Feshbach resonance, and what you have in a Fano Feshbach resonance is two atoms can scatter off each other. This is the continuum. This would be your broad feature. But then they can also scatter through a molecular state, and this is a narrow feature. And what we've identified now for electromagnetically induced transparency are the two features. One is the single photon absorption, one is the Ramon resonance.

But in general, the concept is much more general. You have a narrow feature, you have a broader feature. It's responsible for scattering two atoms or it's responsible for scattering a photon, and once the photon or the atoms have been scattered, you have no way of telling which intermediate state was involved. And therefore, you get interference. And what I just said, what is EIT for light is the zero crossing of a scattering length that the atoms do not scatter off each other because the two different processes completely destructively interfere.

So what I've shown here is two Lorentzian, two absorption features. Let me know re-plot it and plot the index of refraction minus 1. Here was EIT, zero detuning. We have a sharp feature here at the two photon resonance, we have a broad feature here at the single photon resonance, and if I now transform the Lorentzian into a dispersive feature. I use freehand, so this is a dispersive feature for the broad transition.

The narrow transition has a much, much sharper dispersive feature, and the important part is now at the EIT, at the detuning delta is zero where we have electromagnetically induced transparency, we have an index of refraction which is exactly one because it's a dark state, you have no absorption, you have no light scattering, you have no reaction to the light. And therefore, the index of refraction of the material is like the index of refraction of the vacuum.
So you have n equals 1. It looks like a vacuum in terms of index of refraction. It looks like a vacuum because you have no absorption. But what you have is you have a large derivative of the index of refraction with the frequency detuning, and that affects, and that’s what I want to tell you now, the group velocity of light.

So anyway, this is maybe as far as I want to push it, and I was actually wondering if this is a little bit too complicated to present in class. But on the other hand, I think it sort of also wraps up the course. We have a three level system, and we find a lot of things we have studied separately before-- two photon Raman feature, single photon light scattering-- but now they act together and they interfere and have this additional feature of electromagnetically induced transparency. Colin?

STUDENT: So the way you drew the level diagram, the f, I think, [INAUDIBLE] state [INAUDIBLE]?

PROFESSOR: Yes. Actually, I emphasized that we have an AC Stark shift, and what I didn’t say when I discussed it here that the AC Stark shift pushes this level down and pushes the other level up. But since we are talking about a very broad resonance in the excited state, for all practical matters, the AC Stark shift doesn’t matter, whereas for the narrow Raman resonance, the AC Stark shift is important.

STUDENT: Also, you showed on the plot that the shift of the excited state [INAUDIBLE].

PROFESSOR: Sorry. Thanks. So the AC Stark shift for that detuning would shift this level up and would shift this level down. What’s the second question?

STUDENT: You drew on the plot--

PROFESSOR: This one?

STUDENT: That one. That the shift of the excited state was much higher than the shift of the ground state.

PROFESSOR: You mean those two shifts?

STUDENT: Yeah.

PROFESSOR: We have to now do the bookkeeping. We have assumed that the coupling laser has a large detuning, the coupling laser is very far away from resonance. And if you want to hit the excited state, we know we need a Raman resonance, which is delta 2, but the Raman resonance, capital delta 2, means that we are smack on the single photon resonance for the probe laser.
So this feature is pretty much we take the ground state and go exactly to the excited state with a single photon.

There is an AC Stark shift involved but it's not relevant here, whereas the other feature is the two photon Raman feature. And the one thing I wanted to point out in this context is that there is actually a small energy splitting between the two photon Raman feature by the AC Stark effect, whereas the EIT feature always happens at Raman resonance delta equals 0.

Because, just to emphasize that, delta equals 0 is really you induce a coherence between g and f. It's a dark state, and when you have a dark state in that situation, you don't have an AC Stark effect. So the EIT feature is at delta equals 0, whereas the photon scattering features, they suffer, or they experience-- it may not be negative to suffer an AC Stark shift, but they experience an AC Stark shift. Further questions? Yes?

**STUDENT:** Can we somehow relate this to Doppler free spectroscopy?

**PROFESSOR:** Can we relate that to Doppler free spectroscopy? Actually, I don't think so because I would say for the whole discussion here, let's assume we have an atom which has infinite mass, which is not moving at all. We're really talking about internal coherences.

However, and that's where it becomes related, if you look at very, very narrow features as a function of detuning then, of course, Doppler shifts play a role, and if you have very, very, very narrow features, you become sensitive to very, very, very small velocities, and therefore, you have an opportunity to cool. So if you can distinguish spectroscopically an atom which moves a tiny little bit and an atom which stands still, if you can, by a narrow line, distinguish the two, then you can actually laser cool this atom.

The EIT feature can give you extremely high resolution. I'm not discussing it here. I've discussed the phenomenon of EIT, which is coherent population trapping. But there is an extension, which is VSCPT, Velocity Selective Coherent Population Trapping, and VSCPT was a powerful method to cool atoms below the recoil limit, but I've not connected anything of coherent population trapping with the Doppler shift. So for all this discussion, please assume the atom is not moving. Other questions?

Then let me just say a few words about the fact that we have a large derivative of the index of refraction, and this, of course, is used for generating what is called slow light. The group velocity of light is the speed of light, but then it has a denominator which is the derivative of the
index of refraction with respect to frequency. Towards the end of the last century, there were predictions that electromagnetically induced transparency would give you very sharp features which can be used for very slow light.

And what eventually triggered major developments in the field was this landmark paper by Lene Hau where she used the Bose-Einstein condensate to eliminate all kinds of Doppler broadening. There are other tricks how you can eliminate it, but this was the most powerful way to just take a Bose-Einstein condensate where atoms have no thermal velocity and, in this research, she was able to show that light propagated at the speed of a bicycle. So it was a dramatic reduction of the speed of light, and this showed the true potential of EIT. There have been other demonstrations before where light has been slowed by a factor of 100 or a few hundred, but eventually, combining that with a Doppler-less feature because the atoms don’t move in a BEC created a dramatic effect.

So we have now two ways how we can get a large derivative, $dn/d\omega$. I've discussed here the general case that we have a narrow feature, a broad feature, because we have the coupling light detuned from the excited state, but let me just point out that even if you have the coupling light on resonance, depends what you really want, but you can get an even stronger feature in the index of refraction versus frequency.

This is now the situation where we have the strong absorption feature, but then we have the EIT window. So we have this superposition of a positive Lorentzian and a negative Lorentzian, and if I now run it through my Kramers-Kronig calculator and I take the dispersive shape, I can sort of do it for the broad feature in this way and for the narrow feature in this way, and now you have to add up the two. And what you realize is at this point, you have a huge $dn/d\omega$.

What I'm plotting here is on the left side, the absorption of the Lorentzian, and you can regard this sharp notch as a second Lorentzian. So you have the positive Lorentzian, negative Lorentzian, and then you take the dispersive features and you add them up with the correct sign. So whether you're realizing now for quite a general situation where you have single photon detuning, which I discussed before, or whether you’re on the single photon resonance, you can have extremely sharp features.

So now you can take it to the next level. You have a light pulse which enters a medium, and now the light pulse slowly moves through the medium. But while the light pulse moves through
the medium, you reduce the strength of the coupling laser. What happens?

So you do now an adiabatic change of your system. You do an adiabatic change of the control field omega 2 while the probe pulse is in the medium. Well, that means that under idealized assumptions which we've discussed, this feature gets narrower and narrower and narrower. If omega 2 goes to 0, the strength of the control field, this feature becomes infinitesimally narrow. And therefore, this feature becomes infinitely sharp, and that means that the group velocity goes to zero. This is now in the popular press, it’s called stopped light or frozen light because the light has come to a standstill.

What really happens is the following. We have our coupling laser, omega 2, and we have our probe laser, omega 1. When we do what I just said is that omega 2 goes through zero, then the dark state originally, for very strong omega 2, remember the dark state was g? But now if we let omega 2 go to zero, the dark state will become f.

And that means that in a way, every photon in the probe pulse has now pumped an atom from g through two photon Raman process into f. So therefore, what it means to stop light or to freeze light means simply that the photons of the laser have turned into an atomic excitation where the excitation is now the state, f. In other words, you have written the photon. The photon has now put the atom into a different hyperfine state.

So if this is done adiabatically, and I can’t do full justice in this course, but this means that the light is coherently converted into the atomic-- when I say "coherently" say "into population," I mean all the quantum phases, everything which was in the quantum nature of the light has now been converted, has been written into the state, f. This is often called, because g and f are hyperfine states, this means that you have coherently converted the photon or the electromagnetic wave in the probe beam into a spin wave or a magnon.

Anyway, I just want to show you the analogy. The fact that you can put the quantum information of light into an atomic state and back and forth, we've discussed that when we had the situation of cavity QED. We prepared a superposition of ground and excited state and exactly the same quantum state which we had in the atom we later found in the cavity as a superposition of the zero photon and the one photon state. So from those general concepts, it should be clear to you that it is possible to coherently transform a quantum state from light to atoms and back to light.

And here you see the different realization. We have a quantum state of the photon in the
probe laser, and we can now describe the excitations in the system in a parametrized way. What it means is for the strong probe laser, for the strong coupling laser, the excitation in the system travels as a photon in the field one, but when you reduce the coupling in the coupling laser, the excitation becomes less and less photon-like. It becomes more and more magnon-like, spin wave like. And the moment you reduce the power in the coupling laser to zero, what used to be an excitation in the electromagnetic field has now been termed adiabatically into a spin excitation. Coherence has been written into the hyperfine states or your atoms, g and f.

All this is done coherently, and therefore reversibly. You can read out the information by simply time reversing the process. You ramp up again the coupling laser, and that adiabatically turns the spin excitation back into an excitation of the electromagnetic field. Any questions?

Well, we have three minutes left, but I’m not getting started with superradiance. We have one more topic left, and this will be the topic on Wednesday, decay superradiance, and this is when we discuss the phenomenon of coherence where we have coherence not only in one atom between two or three levels. We will then discuss on Wednesday if we have coherence between many atoms, and this at the heart of superradiance. OK. See you on Wednesday.