1. Consider a tight-binding model on a lattice with hopping matrix element \( t \). Add an on-site disorder potential \( V_i \), where \( V_i \) is a random variable distributed uniformly between \( \pm \frac{W}{2} \). Consider the one-dimensional case with \( N \) sites.

(a) For a given realization of \( V_i \), consider the eigenvalues for periodic boundary conditions, i.e., \( V_N = V_1 \) and \( \psi_N = \psi_1 \) where \( \psi_1 \) is the wavefunction on site \( i \). The eigenvalues are solved by diagonalizing an \( (N-1) \times (N-1) \) matrix. Set up the form of the matrix.

(b) Now consider a twisted boundary condition, i.e., \( V_N = V_1 \) and \( \psi_N = \psi_1 e^{i\phi} \). How is the matrix modified from (a).

(c) Show that the eigenvalues \( E_\alpha \) in (b) are equivalent to a problem with complex hopping, i.e., \( t \) is replaced by \( t e^{i(\phi/(N-1))} \) and with periodic boundary conditions. This is the problem of a ring with \( N - 1 \) sites with a magnetic flux through the ring. What is the value of the flux in units of the flux quantum \( \hbar c/e \).

(d) Diagonalize the matrix numerically for a given realization of disorder. Choose \( W/t = 2.0 \) and \( N = 20 \). Plot the energies of the 10 levels near \( E = 0 \) as a function of \( \phi \). Now increase \( N \) and observe how the picture changes.

(e) For the values of \( W/t \) chosen in part (d) calculate the dimensionless conductance of the sample using the Thouless formula

\[
G = \frac{E_T}{\Delta}
\]

where

\[
E_T = \frac{d^2 E_\alpha}{d\phi^2}
\]

and \( \Delta \) is the average energy level spacing. Calculate \( E_T \) and \( \Delta \) by averaging over the 10 levels near \( E = 0 \) and by averaging over a number of realizations of the random potential. Check the dependence of \( G \) as a function of sample size \( N \).
(f) Optional. If you are interested, you may repeat the problem for a two-dimensional square lattice and contrast the behavior. Compare $W/t = 4$ and 9.